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Class AK-manifold of Concircular curvature tensor (V)

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ABSTRACT

T he current deals with new Three classes of almost Kahler manifold W2 of Concircular curvature tensor are Calculating differential geometrical but also topological parameters appropriate for new classes $\overline{V}1$, $\overline{V}2$, and $\overline{V}3$, are the focus of the paper. Through it, an equivalence relationship was obtained between these classes and one of or more the Tensor compound in the adjoint G-structure space and then construct a relation between this new classes.

Introduction

Three different kinds of almost Hermitian manifolds, each of which is defined in terms of the Riemannian curvature tensor, were established by A.Gray [1].These classes are designated as R_1 , R_2 , and R_3 . The class R_1 stipulates what a parakahler manifold [2]. The R_3 class includes RK-manifolds [3].The identities of R_1 , R_2 , and R_3 were demonstrated by A. Gray [1],[4] and [5] to the fundamental concept for comprehending the differential-geometrical properties of Kahler manifolds. Following are the components that make up the Riemannian curvature tensor:

 $\begin{array}{l} R_1{:}{<}\;R(\kappa_1,\kappa_2)\;\kappa_3,\kappa_4{>} = {<}R(J\kappa_1,J\kappa_2)\;\kappa_3,\kappa_4{>}\;;\\ R_2{:}{<}\;\;R(\kappa_1,\kappa_2)\;\;\kappa_3,\kappa_4{>} = {<}R(J\kappa_1,J\kappa_2)\;\;\kappa_3,\kappa_4{>}\;\;+{<}\\ R(J\kappa_1,\kappa_2)\;J\kappa_3,\kappa_4{>}\;\;+{<}R(J\kappa_1,\kappa_2)\;\kappa_3,J\kappa_4{>}\\ R_3{:}{<}\;R(\kappa_1,\kappa_2)\;\kappa_3,\kappa_4{>} = {<}R(J\kappa_1,J\kappa_2)\;J\kappa_3,J\kappa_4{>}\\ \text{The AH-structures belonging to the class }R_i\;\text{have a} \end{array}$

The AH-structures belonging to the class R_i have a tensor R that fulfills the identity $R_i.$ If AH-any subclass of H-structures is named $\cap R_i$ =0, where i is 1, 2, or 3, then it exists. [5] . It is common knowledge that V, $R_1,\ R_2,\ R_3$ [6]. As a result, it makes sense to expect that the manifold class, R_1 manifold class, and lastly the manifold of class R_3 are among the AH-

manifolds that are closest to the Kahler manifold class for differential - geometrical and topological properties. AH -structures, Concircular tensor (V) which satisfies to identity V_i , are referred to as the structures. of class V_i . If $\theta \subset AH$ - any sub class of AH-structures designations \cap R_i = s where i= 1, 2, 3 well - known that $V \subset V_1 \subset V_2 \subset V_3$ [4]. In light of this, it makes sense to anticipate that the H-manifolds with the closest geometrical and topological features will be the ones to the Vaishman - Gray manifold class, manifold class V_1 , manifold class V_2 , but rather finally, manifold of class V_3 . In this paper, we will generalize these relationships, definitions and theories related to them for almost Kahler manifold W_2 of Concircular curvature tensor

Preliminaries

Assuming M is a smooth manifold of size 2n, $C^{\infty}(M)$ is represents an algebra of smooth functions on M, and X(M) is really a module of smooth vector fields on M. The following assumes that all objects are of class $C^{\infty}(M)$ and include manifold, tensor fields, Therefore, J-almost complex structure (J^{2} =id) on M, g=<.,.> and Almost Hermition (is short, AH)

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structure on a manifold M the couple (J, g). pseudo metric Riemannian on M. In this instance $\langle J9,J\alpha \rangle = \langle 9,\alpha \rangle : 9,\alpha \in X(M)$.

Definition 1 [4]

If the basic form Ω (τ , μ) = $<\tau$, $J\mu>$ closed i. e d Ω = 0, the Hermition manifold is said to be approximately < M, J, g =. ,.> has an almost Kahler structure (AK structure). A manifold that is smooth M with an AK- chassis is referred to as a roughly Kahler manifold (AK-manifold).

Definition 2 [3]

A class manifold is denoted by the letters $(M,\ J,\ g)$ such that :

1) \overline{V}_1 if $< V (\kappa_1, \kappa_2) \kappa_3$, $\kappa_4 > = < V(\kappa_1, \kappa_2) J\kappa_3$, $J\kappa_4 >$; 2) \overline{V}_2 if $< V(\kappa_1, \kappa_2) \kappa_3, \kappa_4 > = < V(J\kappa_1, J\kappa_2) \kappa_3, \kappa_4 > + < V(J\kappa_1, \kappa_2) J\kappa_3, \kappa_4 > + < V(J\kappa_1, \kappa_2) J\kappa_3, \kappa_4 > + < V(J\kappa_1, \kappa_2) J\kappa_3, J\kappa_4 > ;$ 3) \overline{V}_3 if $< V(\kappa_1, \kappa_2) \kappa_3, \kappa_4 > = < V(J\kappa_1, J\kappa_2) J\kappa_3, J\kappa_4 > ;$

Note3:

From history theorem which states ((The following are our inclusion relationships: i) $V_0 = V_3$, ii) $V_1 = V_2$, iii) $V_4 = V_7$, iv) $V_5 = V_6$)).

We follows that AK-manifold of class $V_0 = V_3 = V_5 = V_6$ are also class \overline{V}_3 manifolds. The meaning of the specified curvature identities of is most obvious when expressed in terms of an a spectrum Concircular curvature tensor.

Theorem 4

Consider $W=(J,\,g=<.\,,\,.>)$ represents almost Kahler manifold .Then the following statements are identical in this case :

1) W denote a class's structure of \bar{V}_3 ;

 $2)V_{(0)} = 0$ and

(3) The identities $V_{bcd}^a = 0$ are reasonable in space of adjont G-structure space.

Proof:

Consider W denote a class's structure of \overline{V}_3 . W without doubt, the same as identity $V(\kappa_1, \kappa_2)$ $\kappa_3 + J$ $V(J\kappa_1, J\kappa_2)$ $J\kappa_3 = 0$; $\kappa_1, \kappa_2, \kappa_3 \in X(M)$

Spectral tensors are defined as follows:

$$\begin{split} &V(\kappa_1,\,\kappa_2)\,\,\kappa_3 = \,V_{(0)}(\kappa_1,\,\kappa_2)\,\,\kappa_3 \, + V_{(1)}(\kappa_1,\,\kappa_2)\,\,\kappa_3 \, + \,V_{(2)}(\kappa_1,\,\kappa_2) \\ &\kappa_2) \quad \kappa_3 \quad + V_{(3)}(\kappa_1,\,\kappa_2) \quad \kappa_3 \quad + V_{(4)}(\kappa_1,\,\kappa_2) \quad \kappa_3 \\ &+ V_{(5)}(\kappa_1,\kappa_2)\kappa_3 + V_{(6)}(\kappa_1,\,\kappa_2) \quad \kappa_3 \\ &+ V_{(6)}(\kappa_1,\kappa_2)\kappa_3 + V_{(6)}(\kappa_1,\,\kappa_2) \quad \kappa_3 \end{split}$$

 $+V_{(7)}(\kappa_1,\!\kappa_2)\kappa_3;\!\kappa_1,\!\kappa_2,\!\kappa_3\in\!X(M)$

 $= V_{(0)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(1)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(2)}(\kappa_1,\ \kappa_2)\ \kappa_3 + \\ V_{(3)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(4)}(\kappa_1,\ \kappa_2)\ \kappa_3 + V_{(5)}(\kappa_1,\ \kappa_2)\ \kappa_3 + \\ V_{(6)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(7)}(\kappa_1,\ \kappa_2)\ \kappa_3\ ;\ \kappa_1,\ \kappa_2,\ \kappa_3\in X(M)$

These identities will be defined as follows:

 $V(\kappa_1, \kappa_2) \kappa_3 + JV(J\kappa_1, J\kappa_2) J\kappa_3 = \{ V_{(0)}(\kappa_1, \kappa_2) \kappa_3 + V_{(3)}(\kappa_1, \kappa_2) \kappa_3 + V_{(5)}(\kappa_1, \kappa_2) \kappa_3 + V_{(6)}(\kappa_1, \kappa_2) \}$

The identity is shaped by resources $V(\kappa_1, \kappa_2)$ $\kappa_3 + J$ $V(J\kappa_1, J\kappa_2)$ $J\kappa_3 = 0$ is equal $V_{(0)}(\kappa_1, \kappa_2)$ $\kappa_3 + V_{(3)}(\kappa_1, \kappa_2)$ $\kappa_3 + V_{(5)}(\kappa_1, \kappa_2)$ $\kappa_3 + V_{(6)}(\kappa_1, \kappa_2)$ κ_3 and this identity is the same as other identities $V_{(0)} = V_{(3)} = V_{(5)} = V_{(6)} = 0$.

The following relations can be derived from the obtained identities on the adjoint G-space: structure's, according to characteristics (3):

$$V^a_{bcd} = V^a_{b\hat{c}\hat{d}} = V^a_{\hat{b}c\hat{d}} = 0.$$

Because of materiality tensor C and its properties (3) received relations that are identical to $V_{bcd}^a = 0$, this mean identity $V_{(0)}(\kappa_1, \kappa_2)$ $\kappa_3 = 0$ According to [5], the exact opposite is true.

Theorem 5

Consider W = (J, g = <., .>) represents almost Kahler structure. Then the following statements are identical in this case:

(1) W denote a class's structure of \bar{V}_2 ;

 $(2)V_{(0)} = V_{(7)} = 0$; and

(3) On the associated *G*-structure identities space $V^a_{bcd} = V^a_{\hat{b}\hat{c}\hat{d}} = 0$ are they reasonable $V^a_{bcd} = V^a_{\hat{b}\hat{c}\hat{d}} = 0$

Proof:

Consider W- structure of a class \overline{V}_2 be the case identity \overline{V}_2 will be copied in the following format.

The identity will compute based on the notion of a spectrum tensor once every one has been assembled:

 $\begin{array}{llll} 3) & V(J\kappa_1,\kappa_2)J\kappa_3 \! = \! V_{(0)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(1)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(1)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(2)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(3)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(4)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(5)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(5)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(6)}(J\kappa_1,\kappa_2)J\kappa_3 \! + \! V_{(7)}(J\kappa_1,\kappa_2)J\kappa_3 \! - \! V_{(0)}(\kappa_1,\;\kappa_2) \\ \kappa_3 & -V_{(1)}(\kappa_1,\;\kappa_2)\;\kappa_3 + V_{(2)}(\kappa_1,\;\kappa_2)\;\kappa_3 + V_{(3)}(\kappa_1,\;\kappa_2)\;\kappa_3 \\ +V_{(4)}(\kappa_1,\;\kappa_2)\;\kappa_3 & +V_{(5)}(\kappa_1,\;\kappa_2)\;\kappa_3 - V_{(6)}(\kappa_1,\;\kappa_2)\;\kappa_3 - V_{(7)}(\kappa_1,\;\kappa_2)\;\kappa_3\;;\;\kappa_1,\;\kappa_2,\;\kappa_3 \in \! X(M) \end{array}$

 $\begin{array}{l} 4) \ \ JV(J\kappa_1,\ \kappa_2)\ \kappa_3 = JV_{(0)}(J\kappa_1,\ \kappa_2)\ \kappa_3 + JV_{(1)}(J\kappa_1,\ \kappa_2)\ \kappa_3 \\ +\ JV_{(2)}(J\kappa_1,\ \kappa_2)\ \kappa_3 + JV_{(3)}(J\kappa_1,\ \kappa_2)\ \kappa_3 + JV_{(4)}(J\kappa_1,\ \kappa_2)\ \kappa_3 \\ +V_{(5)}(J\kappa_1,\ \kappa_2)\ \kappa_3 +\ JV_{(6)}(J\kappa_1,\ \kappa_2)\ \kappa_3 +\ JV_{(7)}(J\kappa_1,\ \kappa_2) \\ \kappa_3 = -\ V_{(0)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(1)}(\kappa_1,\ \kappa_2)\ \kappa_3 +\ V_{(2)}(\kappa_1,\ \kappa_2)\ \kappa_3 \\ +V_{(3)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(4)}(\kappa_1,\ \kappa_2)\ \kappa_3 - V_{(5)}(\kappa_1,\ \kappa_2)\ \kappa_3 + V_{(6)}(\kappa_1,\ \kappa_2)\ \kappa_3 + V_{(7)}(\kappa_1,\ \kappa_2)\ \kappa_3 ;\ \kappa_1,\ \kappa_2,\ \kappa_3 \in X(M) \end{array}$

If we replace these decompositions in the previous equality, we get : $V(\kappa_1, \, \kappa_2) \, \kappa_3 - V(J\kappa_1, \, J\kappa_2) \, \kappa_3 - V(J\kappa_1, \, \kappa_2) \, J\kappa_3 + JV(J\kappa_1, \, \kappa_2) \, \kappa_3 = 2\{ \, V_{(0)}(\kappa_1, \, \kappa_2) \, \kappa_3 + V_{(3)}(\kappa_1, \, \kappa_2) \, \kappa_3 - V_{(5)}(\kappa_1, \, \kappa_2) \, \kappa_3 + V_{(6)}(\kappa_1, \, \kappa_2) \, \kappa_3 + V_{(7)}(\kappa_1, \, \kappa_2) \, \kappa_3 \}$

This identity is equivalent to that

 $V_{bcd}^{a'} = V_{b\hat{c}\hat{d}}^{a} = V_{b\hat{c}\hat{d}}^{a'} = V_{b\hat{c}\hat{d}}^{a} = V_{b\hat{c}\hat{d}}^{a} = 0.$

Additionally, these identities in the adjacent G-space structures are identical to those in the adjacent G-space structures.

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The received relations are equal to relations due to the materiality tensor V and by history characteristics we have $V^a_{bcd} = V^a_{b\hat{c}\hat{d}} = 0$ i.e. to identities $V_{(0)}(\kappa_1, \, \kappa_2)$ $\kappa_3 = V_{(7)}(\kappa_1, \, \kappa_2)$ κ_3 . Allow for AK's several identities once more $V_{(0)}(\kappa_1, \, \kappa_2)$ $\kappa_3 = V_{(7)}(\kappa_1, \, \kappa_2)$ $\kappa_3 = 0$ are executed.

Then by history from we have: $V(\kappa_1, \kappa_2) \kappa_3 - V(\kappa_1, J\kappa_2) J\kappa_3 - V(J\kappa_1, \kappa_2) J\kappa_3 - V(J\kappa_1, J\kappa_2) \kappa_3 = 0$; i.e. $V(\kappa_1, \kappa_2) \kappa_3 = V(\kappa_1, J\kappa_2) J\kappa_3 + V(J\kappa_1, \kappa_2) J\kappa_3 + V(J\kappa_1, J\kappa_2) \kappa_3$

In the received identity instead of $V(\kappa_1, J\kappa_2) J\kappa_3$ we shall put the value received history from replacement $\kappa_2 \rightarrow J\kappa_2$ and $\kappa_3 \rightarrow J\kappa_3$, i.e

 $V(\kappa_1, J\kappa_2) J\kappa_3 = -JV(J\kappa_1, \kappa_2) \kappa_3$

Then

 $V(\kappa_1,~\kappa_2)~\kappa_3=V(J\kappa_1,~J\kappa_2)~\kappa_3+V(J\kappa_1,~\kappa_2)~J\kappa_3$ - $JV(J\kappa_1,~J\kappa_2)~\kappa_3$ i.e.

 $< V(\kappa_1, \kappa_2) \ \kappa_3, \kappa_4> = < V(J\kappa_1 \ , \ J\kappa_2) \ \kappa_3, \kappa_4> + < V(J\kappa_1, \kappa_2) \ J\kappa_3, \ \kappa_4> + < V(J\kappa_1, \kappa_2) \ \kappa_3 \ , \ J\kappa_4>$ The outcome is that the identical requirement is met by the manifold \overline{V}_2 .

In a similar manner, the next theorem is demonstrated.

Theorem 6

Consider $W=(J,\,g=<\,.\,,\,.>\,)$ represents almost Kahler manifold .Then the following statements are identical in this:

- (1) W denote a class's structure of \bar{V}_1
- (2) $V_{(0)} = V_{(4)} = V_{(7)} = 0;$
- (3) associated identities on the G-structure space $V^a_{bcd} = V^a_{\hat{b}\hat{c}\hat{d}} = V^a_{\hat{b}\hat{c}\hat{d}}$ are reasonable.

<u>proof</u> :

Take, for example, a class's W-structure \overline{V}_1 . It's obvious that it's the same as identity $< V (\kappa_1, \kappa_2) \kappa_3$,

References

- [1] Gray A."Curvature Identilies for Hermitian and Almost Hermitian Manifold " J .diff. geom., V.4,P.283-309,1970.
- [2] Gray A. ,"Curvature identites for Hermitian and Almost Hermitian Manifold " Tohoku Math J.28,NO-4,pp.601-601,1976.
- [3] Gray A., Vanhecke L., "Almost Hermitian Manifold with Constant Holomorphic Sectional curvature", Cast .Pestov. Mat.Vol., NO-12,PP.170-179,1979.
- [4] Gray A. and Hervella L.M. "Sixteen classes of almost Hermitian manifold and their linear invariants "Ann. Math .Pure and Appl.,Vol.123, No.3, pp.35-58,1980.

 $\kappa_4>=<\!V(\kappa_1,\ \kappa_2)\ J\kappa_3$, $J\kappa_4>$ as well as we have $V(\kappa_1,\ \kappa_2)\ \kappa_3+J\ V(\kappa_1,\ \kappa_2)\ J\kappa_3=0$; $\kappa_1,\ \kappa_2,\ \kappa_3\in\!X(M)$ Spectral tensors are through definition:

- $\begin{array}{l} 1) \ V(\kappa_1,\ \kappa_2) \ \kappa_3 = \ V_{(0)}(\kappa_1,\ \kappa_2) \ \kappa_3 \ + V_{(1)}(\kappa_1,\ \kappa_2) \ \kappa_3 \ + \\ V_{(2)}(\kappa_1,\ \kappa_2) \ \kappa_3 \ + V_{(3)}(\kappa_1,\ \kappa_2) \ \kappa_3 \ + V_{(4)}(\kappa_1,\ \kappa_2) \ \kappa_3 \\ + V_{(5)}(\kappa_1,\ \kappa_2) \ \kappa_3 + V_{(6)}(\kappa_1,\ \kappa_2) \ \kappa_3 + V_{(7)}(\kappa_1,\ \kappa_2) \ \kappa_3 \ ; \ \kappa_1, \\ \kappa_2,\ \kappa_3 \in X(M) \end{array}$
- $= -V_{(0)}(\kappa_1, \ \kappa_2) \ \kappa_3 V_{(1)}(\kappa_1, \ \kappa_2) \ \kappa_3 V_{(2)}(\kappa_1, \ \kappa_2) \ \kappa_3 V_{(3)}(\kappa_1, \ \kappa_2) \ \kappa_3 + V_{(4)}(\kappa_1, \ \kappa_2) \ \kappa_3 V_{(5)}(\kappa_1, \ \kappa_2) \ \kappa_3 V_{(6)}(\kappa_1, \ \kappa_2) \ \kappa_3 + V_{(7)}(\kappa_1, \ \kappa_2) \ \kappa_3 \ ; \ \kappa_1, \ \kappa_2, \ \kappa_3 \in X(M) \ .$ Putting (1) and (2) in

This identity is also equivalent to other identities $V_{(0)} = V_{(4)} = V_{(7)} = 0$. So according their history properties the adjoint G- structure's received identities in space are identical to relation $V_{bcd}^a = V_{bcd}^a = V_{b\hat{c}\hat{d}}^a = 0$.

Theorem 7

Consider W = (J , g = < . , . >) is an AK- structure, then the next class insertion $\bar{V}_1 \subset \bar{V}_2 \subset \bar{V}_3$ are resonable.

Proof:

Let's say a class's W-structure is \bar{V}_1 . By history theory, it is identical to $\bar{V}_0 = \bar{V}_4 = \bar{V}_7 = 0$.

As a result of history theorem class $\bar{V}_0 = \bar{V}_7 = 0$, is identical to class \bar{V}_2 . Then $\bar{V}_1 \subset \bar{V}_2$. Furthermore, the class \bar{V}_3 is the same as class \bar{V}_0 . As shown by history theorem so $\bar{V}_1 \subset \bar{V}_2 \subset \bar{V}_3$

- [5] Kirichenko V.F. Rustanoy A.R , Shihab A.A, "On Geometry of the tensor of Conharmonic Curvature of almost Hermitian manifold" Mathmetical notes ,Moscow,Vol.90,No-1,pp,2011.
- [6] Lichnerowicz A., "Theories Globale Des connexionset Des Group D'holonomie", Roma Edizionicremonese, 1955.
- [7] Mileva P "Locally Conformally Kähler Manifolds of Constant type and J-invariant Curvature Tensor", Factauniversitatis Mechanics, Automatic control . 791.804,2003
- [8] Rawah. A. Z "Concircular Curvature Tensor of Nearly Kahler manifold" University of Tikrit, Collage of Education, 2015.



صفوف تنزرالانحناء الهرمتي الدائري لمنطوي كوهلر التقريبي

ياسين خضير عباس , علي عبد المجيد شهاب

الملخص

تم تعریف ثلاث فئات جدیدة من منطوی کوهلر التقریبی من تنزر الانحناء الهرمتی الدائری , ویهدف هذا البحث الی حساب الخصائص التفاضلیة $V_1, \overline{V}_2, \overline{V}_3$ والتی من خلالها تم الحصول علی علاقة تکافؤ بین واحدة من هذه الفئات او اکثر من مرکبات تنزر الانحناء الهرمتی الدائری لمنطوی کوهلر التقریبی , واخیرا تم ایجاد علاقة بین V_1, V_2, V_3 ومع بعضها البعض .