Tikrit Journal of Pure Science Vol. 26 (2) 2021

TJPS

IASJ

المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

Optimization of Interval Type-2 Fuzzy Logic System By using A New Hybrid Method of Whale Optimization algorithm and Extreme Learning Machine

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https://doi.org/10.25130/tjps.v26i2.129

ARTICLE INFO.

Article history:

-Received: 22 / 12 / 2020

-Accepted: 16 / 2 / 2021

-Available online: / / 2021

Keywords: Interval type 2 Fuzzy logic-system, Type 1 fuzzy logic, Whale Optimization algorithm, Optimization, Extreme Learning Machine.

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ABSTRACT

The problem of searching for the best values of the fuzzy logic parameters (T1FLS) is consider complex problems, and for type-2 fuzzy logic system (T2FLS) the problem is more complex, in special case interval type-2 fuzzy logic system (IT2FLS). The Researchers have used many methods and algorithms to solve this problem, and among the most important algorithms used in this field are the(Meta-heuristic) algorithms. Because Meta-heuristic algorithms have a high capacity in the practical field, so we used one of the modern algorithms in this field, which is the Whale Optimization algorithm (WOA). We are used the (WOA) algorithm together with the Extreme Learning Machine (ELM) algorithm as a hybrid algorithm to find the best parameters for the IT2FLS. Whereas, the (WOA) algorithm was used to estimate the values of the antecedent for the system, and the (ELM) algorithm was used to find the values of the consequent parts in the system. The simulation results show that the proposed algorithm is effective for a system (IT2FLS).

1. Introduction

Fuzzy logic systems have been successfully applied to wide range of problems in different application area. The type-1 fuzzy logic approach faces problems when confronted with dynamic environments containing some types of uncertainty found in large number of real-world application. All these doubts translate into doubts about membership function. Type1 fuzzy logic cannot fully deal with these uncertainties because type-1 fuzzy logic is precise in nature and for many applications it is unable to model knowledge adequately where type-2 fuzzy logic offers a higher level of imprecision. Although fuzzy logic type-2 is growing topic of research with plenty of evidence for successful application [5]. Modeling and Control is the most used application of both Type 1 (FLS), and type-2 (IT2) FLS separator. Basically, FLS implements a function that mapping inputs and outputs. In many cases, a continuous and smooth layout of the input and output of FLS is required, since most physical systems are continuous, and a continuous and smooth control surface is usually more suitable in terms of stability and performance, for example, Wu, Tan and Jami Etal. Showed that the IT2FL controller may out perform its T1FL analogs because it gives a smoother control surface, especially in steady-state surroundings (Both error approach and change error 0) [21]. The notion optimization is fundamental in every stage of our daily life. The desire to improve or be the best in almost every area. In engineering, for example, we want to get the best results with the resources available. In an increasingly competitive world, we cannot simple be satisfied "only satisfactory" solution/performance but instead we have looking forward to designing a better system. While new product field design: aerospace, agriculture, automotive, biomedicine, electrical, chemical, etc. , We must use design tools which provide the desired results in a timely and economical. Improving the area has received much attention in recent years; this is mainly due to rapid advances in computer technology, including the development and availability of easy-to-use software, high-speed and parallel processors, and artificial neural network. The optimization process includes creating a suitable model. Modeling is a mathematical procedure to define and express the goal, variable, and limitations of a problem. The goal is a quantitative measure of performance of the system to be minimized or maximized. Variables are the components of the system whose value can be found. The constraint is a condition of the improvement problem that the solution must fulfill [2]. Rules-based Fuzzy Inference Systems (FISs), Although every FIS it has a cognitive representation structure in IF-THEN Fuzzy rule formula, FIS insufficient capacity to adapt to changing external environment. Thus, learning concepts of the neural network integrated with fuzzy inference systems, resulting in Fuzzy neural modeling[12].. In this paper, the hybrid of the extreme learning machine (ELM) and Whale optimizer (WOA) were used to optimize the consequent and antecedent parameters, respectively. The idea of the (ELM) is based on idea of square errors between the actual and approximate value. Meta-heuristic optimization algorithms have become popular in engineering applications for:

1- The are based on concepts that are fairly simple and easy to implement; 2- Dose not require gradient information; 3- It can exceed the domestic optima; 4- It can be widely used in different specialties. The search process in Population-based meta-heuristic optimization algorithms is divided into two stages; Exploration and exploitation [18]. In this paper, we used a new meta-optimization algorithm (i.e. whale optimization algorithm, WOA) that simulates humpback whaling behavior. This paper is divided into the following parts: **Section 2:** Describe the fuzzy logic, fuzzy sets, IT2FL. **Section 3**: Describe the ELM, and WOA. **Section 4**: It demonstrates the practical side of applying a hybrid-learning algorithm and the whale optimizer**. Section 5**: Presents the experimental results and analysis, and we are provide concluding observation.

2. Fuzzy sets

The theory of fuzzy logic is a generalization of classical logic (crisp logic) theory, meaning that the classical logic is special case of the fuzzy logic as shown in Fig.1 below:

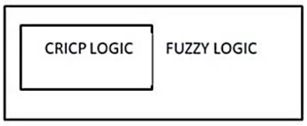

Fig.1 The classical set theory is a subset of the theory of fuzzy sets.

Definition 1. Type-1 fuzzy set[12,15].

Let X is universe of discourse (non-empty set), The fuzzy set B in X is defined as an ordered set pair, as shown below:

$B = \{(x, \mu_B(x)) : x \in X\} \ldots$ (1)

where μ_B , it is called a Membership function (MF for short) type-1 for fuzzy set B , and $\mu_B(x)$ represent the degree of membership of x to B, and $\forall\, x \in X$. In which $0 \le \mu_B(x) \le 1$ as for Crisp set B, each element either belongs or does not belong to the set B, and the membership function mathematically can be expressed as follows:

$$\mu_B(x) = \begin{cases} 1 & \text{if } x \in B \\ 0 & \text{if } x \notin B \end{cases} \quad \ldots (2)$$

The figure 2 a, b shows Membership function in crisp and Fuzzy set, respectively:

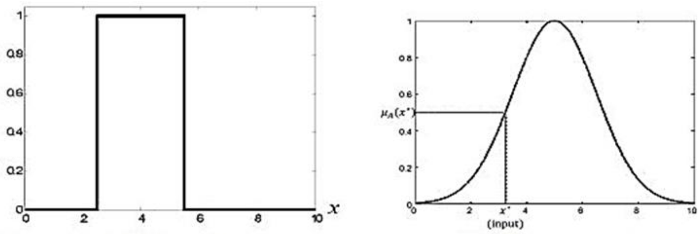

Fig.2 a: MF for crisp set b: MF for fuzzy set

Fuzzy set B can be written as follows:

If universe X is continuous then $B = \int_{x \in X} \mu_B(x)/x$ …. (3)

If universe X is discrete then $B = \sum_{x \in X_d} \mu_B(x)/x$ … (4)

Definition 2. Type2 Fuzzy set, let X be non-empty set, a type2 Fuzzy set (T2FS) $\tilde{B}$ is

$\tilde{B} = \{(x, v), \mu_{\tilde{B}}(x, v) : x \in X, v \in V\} \ldots$ (5)

where v is primary membership of x in $\tilde{B}$ and $\mu_{\tilde{B}} : X \times V \to [0,1]$, hence $\mu_{\tilde{B}}(x, v)$ represent the degree of membership of (x, v) ,and the T2FS can be written as :

$$\tilde{B} = \begin{cases} \int_{x \in X} \int_{v \in V} \mu_{\tilde{B}}(x, v)/(x, v) & \text{if } X \text{ is continuous} . \\ \sum_{x \in X_d} \sum_{v \in V d} \mu_{\tilde{B}}(x, v)/(x, v) & \text{if } X \text{ is discrete} . \end{cases}$$

….(6)

Where $\int\int$ and $\sum\sum$ denotes the union over all admissible point (x, v) in the domain $X \times V$ or $X_d \times V_d$ [14].

2.1 Type 2fuzzy logic

Fuzzy logic of type2 was established in 1975 by the scientist Lotfi Zadeh [6], as a developed system for fuzzy logic of first type (T1FL), this system was found to fill the gaps in the first model. That is, in the sense of fogging fuzzy by specifying a period of membership degrees for each input value, and through it the concept of the area of uncertainty is formed, as shown in the following figure 3:

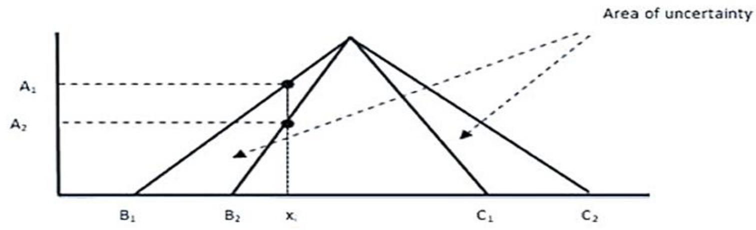

Fig.3 Draw the membership function in periods.

Where [B_1, B2], [C_1,C_2] are represent the parameters of the membership function are in the form of a period and between them lies a region and [A_1,A_2] represent membership degrees of X values and no single value as in (T1FL). Unlike the (T1FL), the value of the degree MF is crisp, and in general formula to (T2FL) is:

$\tilde{B}$={(x , v), $\mu_{\tilde{B}}(x,v)$:∀ $x \in X$,∀$v \in J_x \subseteq$ [0,1] } …. (7)

where x is primary domain, and J_x is secondary domain[16] .

Definition 3. The support of $\tilde{B}$, which also called the domain of uncertainty of $\tilde{B}$,

[DOU($\tilde{A}$) for short] , Consisting of all pairs (x , v) in $X \times [0,1]$ So that$\mu_{\tilde{B}}(x,v)>0$ and defined as :

DOU($\tilde{B}$) =(x, v) ∈ $X \times [0,1]$: $\mu_{\tilde{B}}(x,v)>0$}= $\cup_{x \in X} J_x$ [13] . …(8)

Unlike the concept **footprint** of uncertainty (FOU), which can be as a special case of (DOU), and note that FOU of (T-2FS$\tilde{B}$) is bounded by tow membership functions as follows

Upper MFS of FOU($\tilde{B}$) is $\overline{\mu}_{\tilde{B}}(x)$.

I.e. $\overline{\mu}_{\tilde{B}}(x)$=sup { v: $v \in$[0, 1] , $\mu_{\tilde{B}}(x,v)>0$}. …. (9)

Lower MFS of FOU($\tilde{B}$) is $\underline{\mu}_B(x)$.

I.e. $\underline{\mu}_{\tilde{B}}(x)$= inf { v: $v \in$[0, 1] , $\mu_{\tilde{B}}(x,v)>0$}. …(10)

And the domain of uncertainty of ($\tilde{B}$) is called the **footprint of uncertainty of $\tilde{B}$** .

I.e. DOU($\tilde{B}$)= FOU($\tilde{B}$) ={(x , v):$x \in X$, and $v \in [\underline{\mu}_B(x), \overline{\mu}_{\tilde{B}}(x)]$} [51]. ….(11)

And the area between $\underline{\mu}_{\tilde{B}}(x)$ and $\overline{\mu}_{\tilde{B}}(x)$ is the footprint of uncertainty (FOU) [4].

2.2 INTERVAL TYPE-2 FUZZY LOGIC System

The degrees of membership in this type deal with two dimension .The first dimension is the domain and represents the elements of X or the input data, and the second dimension represents the range and the values of degrees of membership, as the values of these degrees that are obtained are membership degrees of the first type (T1FL), and the grades are called primary membership grades, and have the same characteristics of (T1FL). The two functions represent an upper function ,lower function, and the area of uncertainty, which is the form of provide that contains with in it degrees that are defined in the form [$\mu_l(x)$, $\mu_u(x)$].The third dimension has no effect in this type because all its degrees are defined by one degree namely(1), which is called secondary membership grades. That is ,in sense of every located within the area of uncertainty called the primary and secondary degrees, a knowledge that is defined by one membership degree, and the general form of these function is as follows:

$\tilde{B}$={(x , v)/1: x∈ X, $\mu(x)$∈[0, 1]} ….(12)

Where x is primary domain, $\mu(x)$ is primary membership [$\mu_l(x)$, $\mu_u(x)$], 1 is secondary membership [12,15]. In general, secondary membership functions of (T2FS) take values in [0, 1]; but when take the value (1) for each (T2S) is called an interval type-2 fuzzy set (IT2FS) then: $\mu_{\tilde{B}}$:$X \times V \rightarrow 1$.

Definition 4. Let $v \in$ [0, 1] and $\mu_{\tilde{B}}(x,v)$=1 for x∈ X, then $\tilde{B}$ is called IT2FS[14]. And IT2FS can be written as:

$$\tilde{B}=\begin{cases} \int_{x\in X}\int_{v\in V} 1/(x,v) & if\ X\ is\ continuous. \\ \sum_{x\in X_d}\sum_{v\in V_d} 1/(x,v) & if\ X_d\ \ is\ discrete. \end{cases}$$
…(13)

Where $\int\int\ and\ \sum\sum$ represent the union over all admissible elements (x,v) in the domain $X \times V$ or $X_d \times V_d$ and an IT2FS is fully described by (DOU), then

$\tilde{B}$= 1/ DOU($\tilde{B}$) …..(14)

2.3. Schematic diagram of an (IT2FLS):

It is possible to summarize the stages of construction of the IT2FLS through the following diagram, and shown in the (figure 4), and it similar to its (IT2FLS)counterpart, the major differences being that at least one of the fuzzy sets in the rule base is an IT2FS , hence the output are (IT2FSs) form of the inference engine. So we need the type-reducer to convert it to (T1FS) before defuzzification for the output to be crisp set [15].

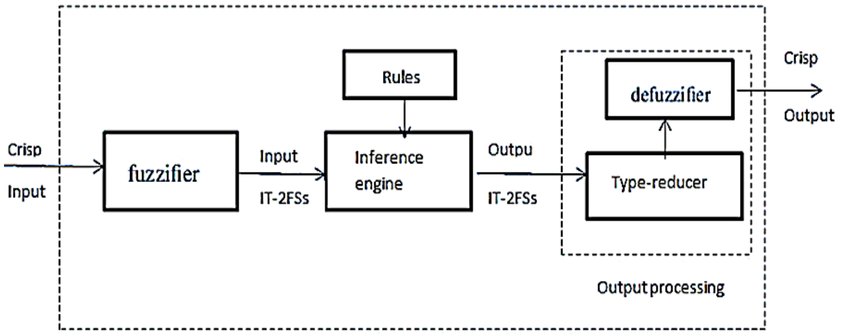

Fig.4 Schematic diagram of an IT2FLS with type- reduction + defuzzification

Now we will explain each part briefly:

2.3.1. Fuzzifier : The fuzzifer is the first phase of the stages and processes of the fuzzy logic throughout this inputs (Crisp) turn out fuzzy input, and for a system there are three type of fuzzifer for (IT2FLS), T-2singleton,T-1non-singleton and IT-2non singleton, unlikely (T1FLS) there are only two types, T-1 singleton[15]. The fuzzification process used is interval single type2 fuzzification and includes mapping from a digital input vector $x \in X$ into an IT2FLS $\tilde{B}$ in X which activates the inference engine. The firing strength for membership functions is interval [$\underline{f}^{\mu}$, $\overline{f^{\mu}}$] , and Gaussian membership function is defined as follows :

$$\overline{\mu_{ik}}\,(x_i) = exp\left(-\frac{(x_i - c_{ik})^2}{2\overline{\sigma}_{ik}^2}\right) \quad ....(15)$$

$$\underline{\mu}_{ik}\,(x_i) = exp\left(-\frac{(x_i - c_{ik})^2}{2\underline{\sigma}_{ik}^2}\right) \quad ....(16)$$

Where c_{ik} denote the mean of Gaussian membership function and σ_{ik} denote the deviation (i.e. width) of Gaussian membership function[9] .

2.3.2. Fuzzy inference engine:

It is the second stage , and it assigns the fuzzy input sets (IT2FSs) into the fuzzy output sets (IT2FSs) through the fuzzy rules (IF-THEN) of the (IT2FS) ,and because of the nature of fuzzy membership function (MFS), which distinguishes between the first and second types of fuzzy systems, it leads to the formation of independent rules and two types of basic structures for the rules, as in the (T1FLS), which are Zadeh rules and TSK. And these rules are called (IT2);in order to distinguish between the two types (T1 and IT2FSs) [15].

The IF-THEN rules of IT-2FLS can be express as follows:

R_k: IF$x_1$is$\tilde{B}_{1k}$and $x_2$is$\tilde{B}_{2k}$…$x_n$is$\tilde{B}_{nk}$Then

$Y_k$is$f(x_1, x_2, \dots, x_n) = w_{0k} + w_{1k}x_1 + \dots + w_{nk}x_n$ …(17)

Where $\tilde{B}_{1k}$, $\tilde{B}_{2k}$, … , $\tilde{B}_{nk}$ are IT-2FS and Y_k, it is the output of the kth a base consisting of a linear combination of the input vector$(x_1, x_2, \dots, x_n)$.

2.3.3. Type-Reduction + Defuzzification:

The output process is the last part of the construction stages of the (IT2FLS), and is divided into two parts as follows:

2.3.4. Type-reduction: It converts or reduces the fuzzy output sets (IT2FSs) resulting from the inference engine into (IT1FSs),and there are many type-reduction , but will use in this search the reduction namely (IT2TSK FLS), and the result after the reduction will be one for any type used, and it is a fuzzy set of the first type (IT1FLS).

2.3.5. Defuzzification:

It is considered the final part of the directing and constructing stages, and defuzzification for the (IT1FS), taken as the average of the resulting endpoint form the type-reduced set [15].

The output of IT2FLS, and the type- reduced interval, computed by Karnik and Mendel (KM algorithms) as$[Y_L, Y_R]$,and type- reduction is based on using the average of Y_L, Y_R. i.e.

$$Y = \frac{Y_L + Y_R}{2} \quad .... (18)$$

where

$$Y_L = \frac{\sum_{k=1}^{l} \overline{f^{\mu}}_k w^k + \sum_{k=l+1}^{m} \underline{f^{\mu}}_k w^k}{\sum_{k=1}^{l} \overline{f^{\mu}}_k + \sum_{k=l+1}^{m} \underline{f^{\mu}}_k} \quad ....(19)$$

$$Y_R = \frac{\sum_{k=1}^{R} \underline{f^{\mu}}_k w^k + \sum_{k=R+1}^{m} \overline{f^{\mu}}_k w^k}{\sum_{k=1}^{R} \underline{f^{\mu}}_k + \sum_{k=R+1}^{m} \overline{f^{\mu}}_k} \quad .... (20)$$

Where $\overline{f^{\mu}}_k$ is upper membership, and $\underline{f^{\mu}}_k$ is lower membership are defined as:

$$\overline{f^{\mu}}_k(x) = \overline{\mu}_{\tilde{B}_{1k}}(x_1) * \overline{\mu}_{\tilde{B}_{2k}}(x_2) * \dots * \overline{\mu}_{\tilde{B}_{nk}}(x_n) \quad ....(21)$$

$$\underline{f^{\mu}}_k(x) = \underline{\mu}_{\tilde{B}_{1k}}(x_1) * \underline{\mu}_{\tilde{B}_{2k}}(x_2) * \dots * \underline{\mu}_{\tilde{B}_{nk}}(x_n) \quad ...(22)$$

The performance index used in the experiment is root mean square error (RMSE) as expressed by equation:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(Y_i^a - Y)^2} \quad \dots (23)$$

Where Y^a is output required,Y is output form, and N is the number of test data points [9,20].

3. Hybrid learning algorithm

The proposed hybrid algorithm is described in this section, learning was used for the extreme learning machine (ELM) and the whale optimizer (WOA) to improve consequents and antecedents parameters of IT2FLS, respectively. The flowchart of WOA-ELM algorithm shown in Fig. 5.

3.1. Extreme learning machine (ELM)

The ELM was proposed by (G-B. Huang) [7] as a method to tune the weight of single hidden layer feed forward neural networks (SLFNs).The standard design includes the weight of hidden layers. To show the mathematical steps of ELM, let we have N learning samples (x_i, y_i), where $x_i = [x_{i1}, x_{i2}, \dots, x_{in}]^T$, $y_i = [y_{i1}, y_{i2}, \dots, y_{im}]^T$, where $i = 1,2,\dots,N$, s.t. $x_i \in R^n$, and, $y_i \in R^m$ .Then the standard SLFN, and the nodes of hidden layer in the activation function $f(x)$ can be expressed as a samples of N with zero error. By other words, we will find the solution of the mean as:

$\sum_{j=1}^{L} \|0_j - y_j\| = 0 \implies \sum_{i=1}^{L} \beta_i \, g_i\left(w_i x_j + b_i\right) = y_i$, where $j = 1,2,\dots,N$

$\implies H\beta = Y$ …..(24)

$$H(w_1, w_2, \dots, b_1, \dots, b_L, x_1, \dots, x_n) = \begin{bmatrix} f(w_1.x_1 + b_1) & \cdots & f(w_L.x_1 + b_L) \\ \vdots & \ddots & \vdots \\ f(w_1.x_n + b_1) & \cdots & f(w_L.x_n + b_L) \end{bmatrix}_{N\times L} \quad \dots(25)$$

$$\implies \beta = \begin{bmatrix} \beta_1^T \\ \beta_2^T \\ \vdots \\ \beta_L^T \end{bmatrix}_{L\times m} , \quad Y = \begin{bmatrix} y_1^T \\ y_2^T \\ \vdots \\ y_N^T \end{bmatrix}_{N\times m} \quad \dots(26)$$

The solution of linear system (24) is find by using the least square solution of the system $\beta = H^{\dagger}$, where $H^{\dagger}$(Pseudo − Inverse) is the Moore-Penrose generalized inverse of the hidden layer output matrix H , because the matrix H is not square or it is singular [11] .

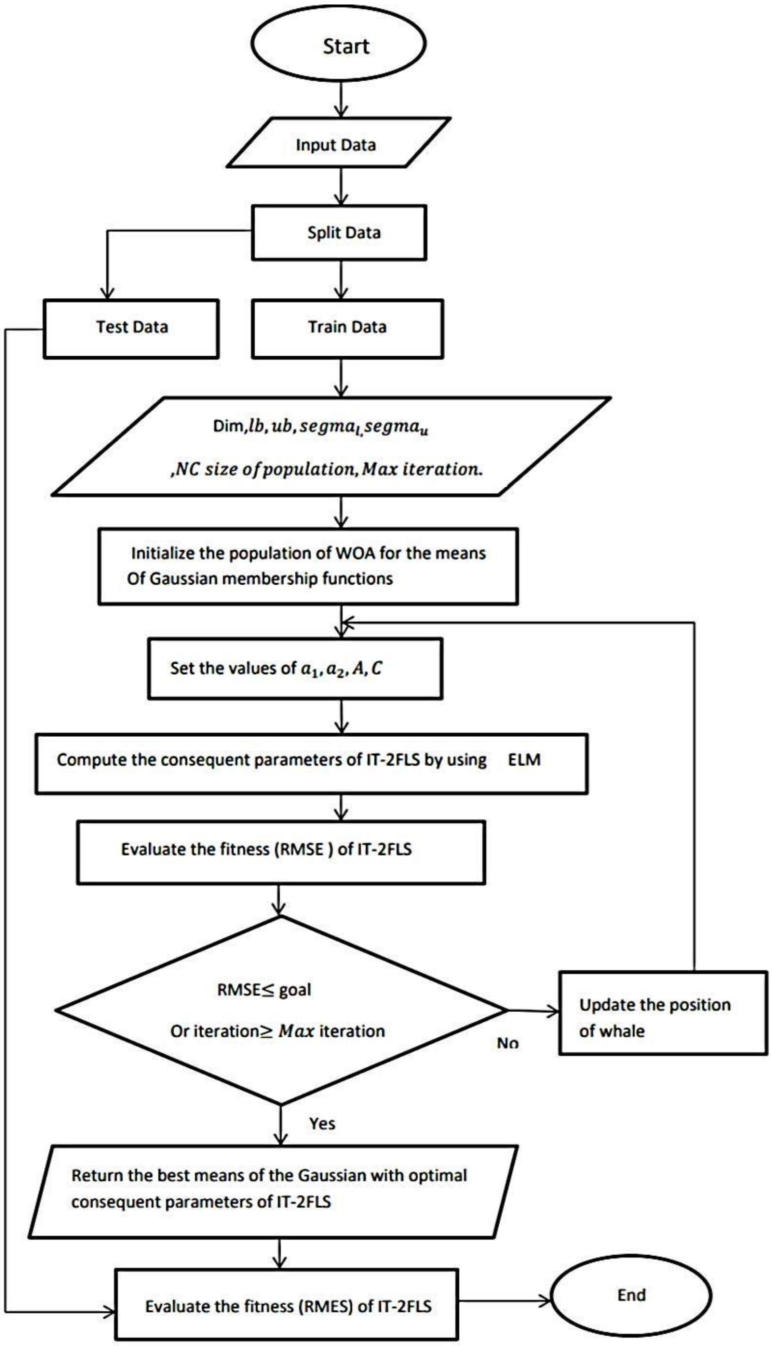

Fig.5: The flowchart of the proposed hybrid algorithm WOA-ELM for IT2FL

3.2. Whale Optimization (WOA)

3.2.1. Suggestive ideas: Whales are fictional beings. It is regarded one of largest mammalian in the world. The length of an adult whale can reach 30 meters and weigh 180 Tons. The whales are mostly predators. They never slept because they had to breathe from the ocean surface. Actually, just half of the brain slept. The enjoyable thing about whales is that they are deem extremely intelligent and lovely animals. Depending Hof and Van Gucht, Own whales hives shared in certain regions of their brains identical to human hives called spindle hives. These cells are accountable for judge, passions, and socially behaviors in humans. To analyze it differently, the spindle hives, it makes us unique form other organisms. Whales have double the number of these cells than mature human and is the major cause for them intelligence. Figure 6 shows these mammals. The amazing at humpback whales is their fishing method. The feeding behavior is called the feeding bubble network. Humpback whales prefer to hunt krill fish near the surface. It has been observed that foraging is performed by making distinct bubbles path in shape of a “9” as in the fig.6. Before 2011, this conduct was examined only based on observation from the surface. Humpback Whales diving in 12 m downs then begin forming a bubble in a spiral about the prey and swim towards the roof. Noticeable to mention here's that feeding by Bubble mesh is a unique behavior, it can only be observed in Humpback Whales [18].

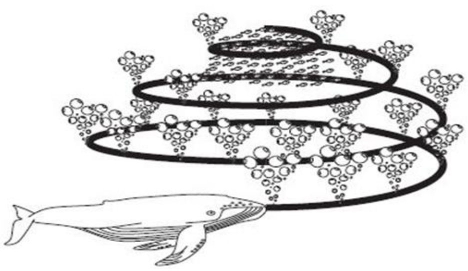

Fig.6: Bubble-net feeding behavior of humpback whales

3.2.2. Mathematical formula of optimization algorithm:

The mathematical formula of prey encirclement, spiral bubble we be maneuverable feeding, and prey hunting first. Then WOA algorithm was suggest.

3.2.2.1 Encirclement -prey:

Humpback Whales surrounding prey can be identified and surround the Prey site. From optimal styling position in the search area it's not previously know, WOA algorithm assume that the currently best candidate solution is the goal prey or near to the optimum. Once that best searching agent is identified, others searching agents will thus try to update their to the best search agent. This conduct is represented by the equation:

$\vec{D}=\left|\vec{c}.\ \overrightarrow{X^*}(t)\ -\vec{x}\ (t)\quad\right|$ ……..(27)

$\vec{x}\ (t+1) = \overrightarrow{X^*}(t)\ \ - \vec{A}\ .\ \vec{D}$ … (28)

Whereas (t) denotes Current redundancy, A and C are coefficients, $\overrightarrow{X^*}$ is the location vector of the better solutions to date, X is the location vector, | | is the absolute value, and(.) is multiplying an item by the element. It should be noted here that $\overrightarrow{X^*}$ has to be refreshed in every iteration if there was a best solution. Vectors A, C are calculated as follows:

$\vec{A}= 2\vec{a}.\ \vec{r} - \vec{a}$ …..(29)

$\vec{C} = 2.\ \vec{r}$ …..(30)

Whereas (a) reduces linearly from 2 to 0 over the course of iteration (Exploration and exploitation), it is a random vector in [0, 1] .Figure 7 demonstrates the rationale behind equation (28) to a two-dimensional problem. Location (x, y) of the searching agent can be refreshed according to the location of the current best register (x^*, y^*). Multiple locations around the best factor with respect to the current location, this can be achieved by modifying the value of vectors A , C. It should be noted that by defining the random vector (r), any position in the search space between the main points shown in fig.7 can be reached. There for, it is equivalent (28). It allows any search agent to update its location near current best solutions and simulate prey encirclement. The same concept can be extended to an n-dimensional search agents will navigate Super cubes about the better solution obtained so far. As aforesaid in the previous part, humpback Whales also attack prey with a bubble network strategies [18].

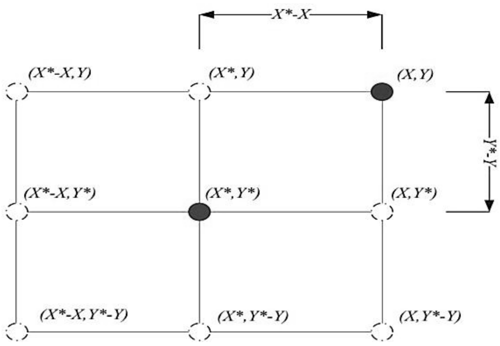

Fig.7: 2D position vectors and their possible next (x^* is the best solution obtained so far).

3.2.2.2. Bubble network attack method (Exploitation)

Because develop a mathematical model of the bubble network conduct of humpback Whales, two methods are designed as follows:

I. Contraction of Encirclement mechanism: This conduct by decrease the value of (a) in parables (28), (29). Notice that volatility of $\vec{A}$ too decreases through (a). To analyze it differently, $\vec{A}$ is a random value in the period [-a, a] whereas (a) is reduced from 2 to 0 through repetitions. Determine random values of $\vec{A}$ in [-1, 1], the new location of the searching agent can be determined everywhere between the original location of the agent and the current better agent. Figure 8(a) appears the possible from(x, y) in the direction of (x^*,

y^*) it can be accomplished by means of 0≤A≤1 in a two- dimensional space.

II. Spiral refreshed location: As they appear in figure 8(b), this method first calculates the distance between the whale when (x, y) and the prey at (x^*, y^*) .Then a spiral equation is formed between the Whale site and the prey to imitation the spiral- style motion of humpback Whales as follows :

$\vec{x}\,(t+1) = \vec{D}\,.\,e^{bl}\,.\cos(2\pi l) + \overrightarrow{X^*}(t)$ ….(31)

Whereas $\vec{D}=\left|\overrightarrow{X^*}(t)\_\vec{x}\,(t)\right|$ .It refers to the distance between the first Whale and the prey (the better, solution obtained so far), (b) it's fixed to determining the shape of the logarithmic helix,(L) is a random number in [-1, 1], and (.) is the multiplication of an item by element. Humpback Whales swimming around prey in a contracting circle and simultaneously along a spiral-style path. To model this concurrent conduct, we suppose there is 50% probability of choosing between either a shrinkage mechanism or a helical model to refreshed the location of Whales. While optimizing.

The mathematical formula is as follows:

$$\vec{x}\,(t+1) = \begin{cases} \overrightarrow{X^*}(t) - \vec{A}\,.\vec{D} & \text{if } p < 0.5 \\ = \vec{D}\,.\,e^{bl}\,.\cos(2\pi l) + \overrightarrow{X^*}(t) & \text{if } p \geq 0.5 \end{cases} \quad \text{.... (32)}$$

Where p is random number in [0, 1].

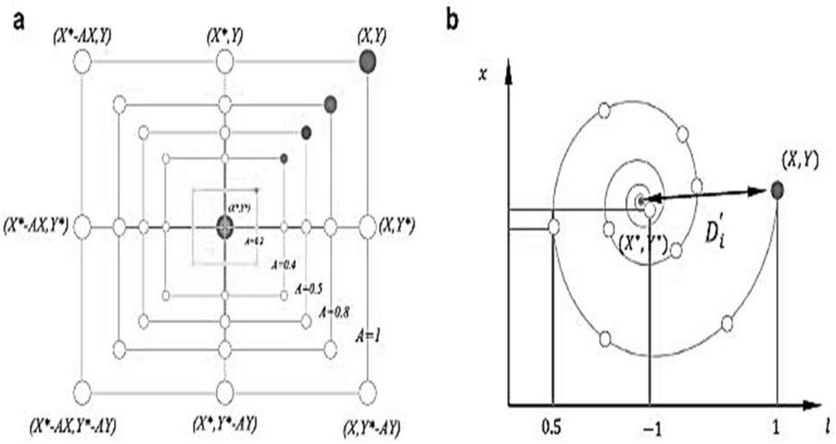

Fig.8: Bubble net search mechanism in WOA (x* is the best solution obtained so far) (a) shrinking encircling mechanism an, (b) spiral updating position

3.3 Searching for the prey (Exploration stage)

The same plan depending on variation of vector A can be applied to search for prey (Exploration). Indeed, humpback Whales search randomly depending on other's position. There for, we used A with random values largest 1 to force the searching agent to stay away from the reference Whale. Unlike the exploitation stage, we update the search agent position in the exploration phase according to randomly selected search agent rather than the better searching agent found so far. This technicality and $|\vec{A}|$ ≥1.Emphasis on Exploration and WOA algorithm for Global search.

The mathematical formula is as follows:

$\vec{D} = \left|\vec{C}\,.\overrightarrow{X_{rand}} - \vec{X}\right|$ ……(33)

$\vec{x}\,(t+1) = \overrightarrow{X_{rand}} - \vec{A}\,.\,\vec{D}$ …..(34)

Where $\overrightarrow{X_{rand}}$ is a random location vector (random Whale) selected from the current population. Figure 9, shows Some potential positions about a specific solution with$|\vec{A}|$ ≥1. WOA algorithm begins with a range of random solutions. In every iteration, the searching agents refreshed their sites with regard to either a randomly selected agent or the best solution we've ever had. The parameter is decrease from 2 to 0 to achieve Exploration and Exploitation, respectively. Random search operator is selection at$|\vec{A}|$ ≥1, but the better solution is determined at $|\vec{A}|$ <1 to update the location of the searching agents. Based on P-value, WOA is capable switch between helical or round motion. Finally, WOA could be considered a global optimization because it involves capable explore / exploit[18] .

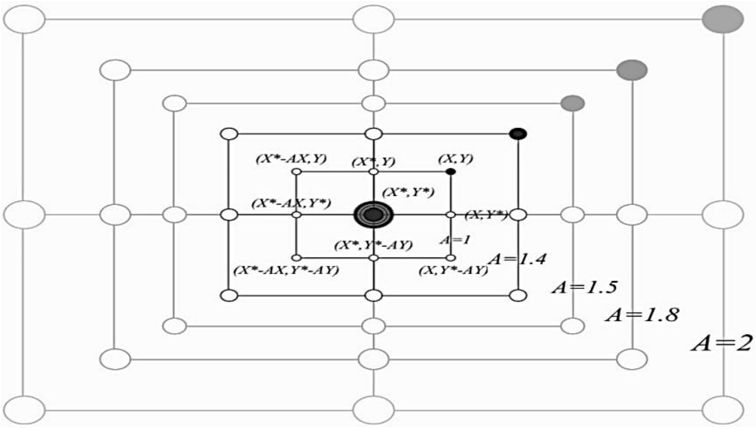

Fig.9: Exploration mechanism implemented in WOA (x* is a randomly chosen search agent).

4. Experiment and Results

In this part, we present our empirical analysis on standard time series and system identification available to the public problems. Data sets and criteria used in carefully selected evaluation (RMSE) for convenience comparison of approach presented herewith current methods. RMSE is defined in equation (23) as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(Y_i^a - Y)^2}$$

Where Y^a is the desired output , Y is the output of the model, and N is the number of the testing data points. Primary values of MF parameters are randomly produced from unit interval [0,1]. If indicators randomly generated M-by-N matrices from unit interval [0,1] for all experiments, whereas (M) is the number of linguistic terms and (N) is the number of rules. Complete experiments were performed by running $MATLAB^{©}$ 2015 running on a 64- Bit Intel(R) Core(TM) i5-7300 CPU@2.50 GHz 2.70 GHz /4 GB RAM configuration computer.

Application for artificially created data sets:

A. System identification problem

The problem of identification involves creating the relationship between system inputs and outputs. The problem is finding such values for the IT2FLS parameters, the structure that the difference between the output$y(k)$ and the identifier output $y_N(k)$ institute the minimum for all input values of $u(k)$. To evaluate the performance of the proposal IT2FLS as an identifier, consideration is given to defining specific inputs and nonlinear outputs. The process is characterized by the following equation of variance [1]:

$$y(k) = u(k)^3 + \frac{y(k-1)}{1+y(k-1)^2} \quad \text{....(35)}$$

whereas $y(k)$ and $y(k-1)$ are current, delayed results are one step, and $u(k)$ is the current entry. While selecting , the output is delayed by one step from the $y(k-1)$ and the control signal $u(k)$ is input IT2FLS as input, and the output of IT2FLS is compared to the factory output $y(k)$. Excitation signal for $y(k)$ in equation (35) is an independent identically distributed standardized sequence over [−1, 1] for about one-fourth of the 400 time steps and sinusoid sin(πk/45) for the residua time. Unknown parameters of IT2FLSWOA are the parameters of the MFs in the second layer (c_{ik}, $\overline{\sigma_{ik}}$, $\underline{\sigma_{ik}}$) and linear function parameters (w and b). As performance standard, root mean square error (RMSE) is used with K = 400.

In figure(10), the RMSE values of IT2FLS with nine Fuzzy rules acquired during learning are viewed, with the number of epochs happening ten. After training, the following test signal is used to find out specific results[1]:

$$u(k) = \begin{cases} -0.7 + \frac{mod(k,50)}{40} & k \le 80 \\ rands(1,1) & 80 < k \le 130 \\ 0.7 + \frac{mod(k,180)}{180} & 130 < k \le 250 \\ 0.6 . \cos(\pi k/50) & k > 250 \end{cases}$$

.......(36)

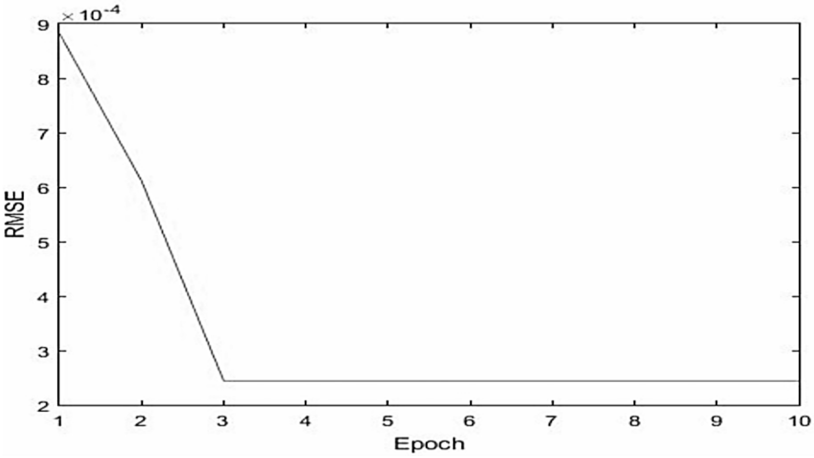

Fig.10: RMSE values obtained during learning.

The RMSE value of the test was obtained in one training period as follows 0.001088 and in ten epochs like 0.000894. Fig.11 appear the Online identification performance of the IT2FLS. Here, the Continuous line is the factory output, and the discontinuous line is the IT2FLS identifier output.

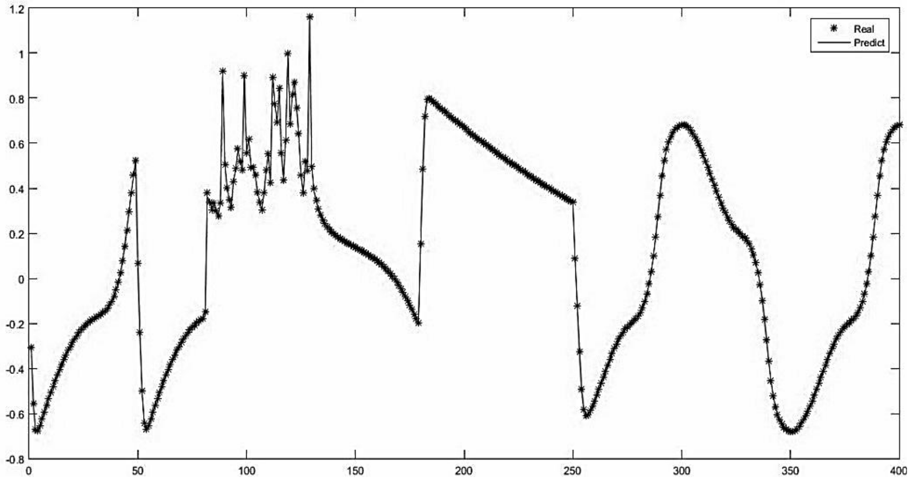

Fig. 11: Simulation results of identification, where the solid line denotes theoutput of the plant, and the dashed line denotes the I2TSK -WOA output.

Table 1: Results of different methods for identification of system .

No.	Methods	RMSE of test
1	Type-2TSK FNS[1]	0.002415
2	Type-2 FNN[22]	0.003204
3	Type-1 FNN[22]	0.006924
4	**IT2FLS-WOA**	**0.001088**

B. Time series Mackey- Glass

To assessment the IT2FLS performance, we applied it for a Mackey-Glass Time Series to the design of the physiological system, It is a well-known data set, known as a nonlinear differential delay equation as follows [6] :

$$\frac{dx(t)}{dt} = \frac{a*x(t-\tau)}{1+x(t-\tau)^n} - b*x(t) \quad .....(37)$$

While a, b and n are fixed values, t is the present time and τ is fixed time delay. We are evaluating the suggested model $\tau = 17$. Identical to [8] , a data set that consists of the 1000 Data points, data points are created running the equation (37). The first 700 data points used for training and the rest 300 used for testing. In order to compare fairly with previous studies, data vector generation is $[x(t-18), x(t-12), x(t-6), x(t); x(t+6)]$ With $x(t+6)$ as the target where $t = 118$ to 1117. A comparison is made between the results IT2FLS trained with WOA and ELM, And its variants are type 2 on Mackey-Glass benchmark dataset. In figure 12, the RMSE values of IT2FLS with nine Fuzzy rules acquired during learning are viewed . Fig.13 shows the current and expected output of Mackey-Glass Time Series using hybrid IT2FLS-WOA. The RMSE value of the test was obtained in one training period as follows 0.005283 and in ten epochs like 0.005284 .The table shows compare results to predict Mackey-Glass time series.

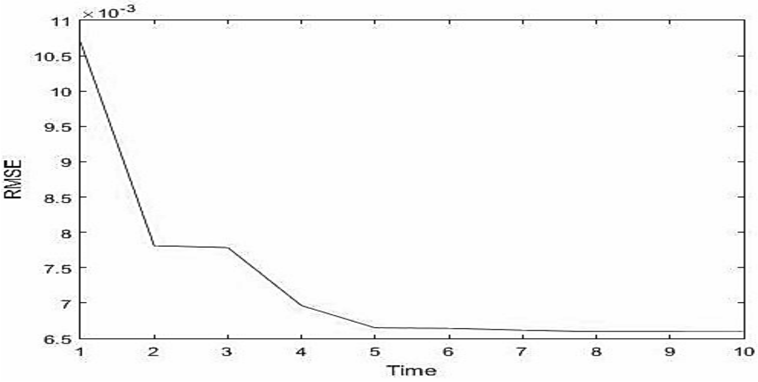

Fig.12. RMSE values obtained during learning.

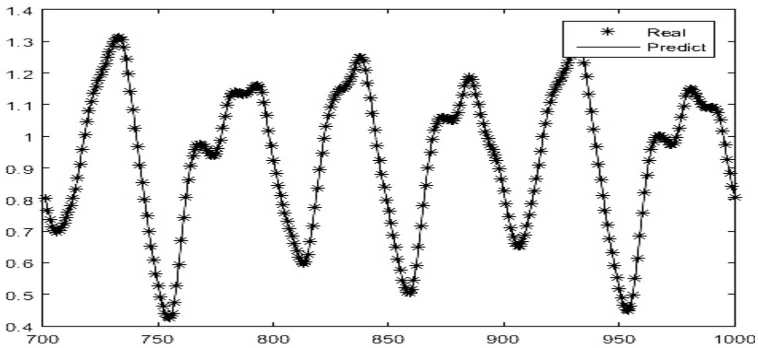

Fig.13: Current, and expected output of Mackey-Glass time series using hybrid IT2FLS-WOA

Table 2: A comparison of results for forecasting Mackey-Glass Ttime Series.

No.	Model	RMSE of test
1	T2-HyFIS-Yager [19]	0.0694
2	MLMVN [3]	0.0056
3	SA-T2FLS [6]	0.0089
4	IFLS-DEKF + GD [10]	0.0054
5	IT2IFLS-DEKF + GD [10]	0.0040
6	**IT2FLS-WOA**	**0.005283**

5. Conclusion

The process of obtaining values various or parameters for the IT2FLS system is a difficult problem in terms of implementation and design. There have been many researches in recent times in this field and most of this research uses hybrid algorithms. The hybridization did not come randomly, but rather because the IT2FLS system consists of two parts of the parameters which are the previous parameters, i.e. the parameters of the membership functions, which are related to the values entered into the system, and the second part includes the consequent parameters, which are the linear functions parameters, which represent the next part (Then) in knowledge rules. In this paper, we used two algorithms for this purpose, namely (WOA) and (ELM) as a hybrid algorithm in order to find the optimal values. Regarding future research, we suggest using the (WOA) algorithm with other algorithms such as the gradient descent algorithm (GD algorithm) or any other algorithm to give good results.

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تحسين تحسين نظام المنطق الضبابي من النوع الثاني ذو الفاصل الزمني باستخدام طريقة هجينة جديدة لخوارزمية تحسين الحوت وآلة التعلم القصوى

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الملخص

تعتبر مشكلة البحث عن أفضل قيم للمعلمات المنطقية الضبابية لـ (T1FLS) من المشكلات المعقدة، وبالنسبة لنظام المنطق الضبابي من النوع الثاني (T2FLS) تكون المشكلة أكثر تعقيدًا ،خصوصا في نظام المنطق الضبابي من النوع الثاني ذو الفاصل الزمني (IT2FLS). واستخدم الباحثون العديد من الطرق والخوارزميات لحل هذه المشكلة، ومن أهم الخوارزميات المستخدمة في هذا المجال هي خوارزميات (Meta-heuristic). نظرًا لأن الخوارزميات الفوقية لها قدرة عالية في المجال العملي، فقد استخدمنا إحدى الخوارزميات الحديثة في هذا المجال، وهي خوارزمية تحسين الحيتان (WOA). نحن نستخدم خوارزمية (WOA) مع خوارزمية Extreme Learning Machine (ELM) كخوارزمية هجينة للعثور على أفضل المعلمات لـ IT2FLS. حيث تم استخدام خوارزمية (WOA) لتقدير قيم سابقة للنظام، وتم استخدام خوارزمية (ELM) لإيجاد قيم الأجزاء الناتجة او اللاحقة في النظام. وتظهر نتائج المحاكاة أن الخوارزمية المقترحة فعالة لنظام (IT2FLS).