



# Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: <http://tjps.tu.edu.iq/index.php/j>



## A Study of Nuclear Structure of $^{96,98}_{42}\text{Mo}$ by Using Interacting Boson Model-1

Berun N. Ghafoor

Department of Physics, College of Education, Sulaimani University, Sulaimani, Iraq, 46001

**Keywords:** Interacting Boson Model-1, Energy levels, B(E2) transitions.

### ARTICLE INFO.

#### Article history:

-Received: 14 June 2023  
-Received in revised form: 14 Aug. 2023  
-Accepted: 15 Aug. 2023  
-Final Proofreading: 24 Dec. 2023  
-Available online: 25 Dec. 2023

#### Corresponding Author\*:

Berun N. Ghafoor

[berun.ghafoor@univsul.edu.iq](mailto:berun.ghafoor@univsul.edu.iq)

©2023 THIS IS AN OPEN ACCESS ARTICLE UNDER THE CC BY LICENSE

<http://creativecommons.org/licenses/by/4.0/>



### ABSTRACT

In this study, the interacting Boson Model (IBM-1) was used to find the best Hamiltonian structure for studying the  $^{96,98}_{42}\text{Mo}$  structure. For generating the Hamiltonian structure, the calculated energy levels (ground, beta, and gamma bands) and B(E2) transitions for  $^{96,98}_{42}\text{Mo}$  were explored using the optimal parameter values. According to this study and the IBM test findings,  $^{96,98}_{42}\text{Mo}$  are stable isotopes of Molybdenum, and their transition behaviors are close to the SU(5) limitations. When comparing the findings to the experimental data, they demonstrated a good agreement.

## دراسة التركيب النووي لـ $^{96,98}_{42}\text{Mo}$ باستخدام نموذج البوزونات المتفاعلة الأول

بيرون نصرالدين غفور

قسم الفيزياء - كلية التربية - جامعة السليمانية - العراق ٢٠٠١

### الملخص

في هذه الدراسة تم استخدام نموذج البوزونات المتفاعلة (IBM-1) لإيجاد أفضل هيكل هاميلتوني لدراسة بنية  $^{96,98}_{42}\text{Mo}$ . لتوليد هاميلتونيان ، تم استكشاف مستويات الطاقة المحسوبة (الأرضي ، بيتا ، وجاما) وانتقالات B (E2) باستخدام أفضل قيم المعلمات المناسبة. وفقاً لهذا البحث ونتائج اختبار IBM ، فإن  $^{96,98}_{42}\text{Mo}$  هي نظائر مستقرة من الموليبدنوم ، وسلوكياتها الانتقالية تقترب من حدود SU(5). عند مقارنة النتائج بالبيانات التجريبية ، فإنها تظهر توافقاً جيداً.

### Introduction

A substantial amount of research has gone into studying the shape and properties of complex nuclei, which has led to the development of several nuclear models. As a result, approximations must be used to grasp the structure of complex atomic nuclei. The structure of the nuclei affects the selection of an appropriate estimate. Arima and Iachello created the Interacting Boson Model (IBM) to study the structure and activity of even-even nuclei. This IBM is based on both the well-known shell model and mathematical group atomic nucleus models [1, 2].

Despite its comparatively basic construction, this model has the potential to be a powerful instrument. Furthermore, it is of great scientific significance because it demonstrates the dynamical symmetries of several nuclei, which are made visible using Lie algebras. The basic concept is that the low energy group degrees of freedom in nuclei can be represented by proton and neutron bosons with spins of 0 and 2. Because the bosons could be couples of holes or particles, the IBM-1's s (L=0) and d (L=2) bosons have six sub-states and thus form a six-dimensional universe. This leads to a definition in terms of the six-dimensional unitary group, U(6). As a consequence, using group-theoretical techniques, many of the IBM-1's distinguishing features can be found and expressed analytically. The primary distinguishing feature of IBM-1 is that both bosons (neutron and proton) are identical. The various reductions of U(6) result in three dynamical symmetries known as SU(5), SU(3), and O(6), which are pertinent to the spherical vibrator, deformed rotor, and asymmetric ( $\gamma$ -soft), in that sequence [3-5].

The current study aims to evaluate the dynamical symmetry, energy levels, B(E2), and quadrupole moment values of  $^{96,98}_{42}\text{Mo}$  within the (IBM-1) context.

### 2. Theoretical Basis of IBM-1

Interacting Boson Model (IBM-1) is a theoretical model used to describe the collective behavior of atomic nuclei. It is based on the assumption that atomic nuclei can be viewed as systems of interacting bosons, which represent pairs of nucleons (protons and neutrons) that are coupled together in the nucleus. In IBM-1, the Hamiltonian structure describes the interactions between pairs of s-bosons and pairs of d-bosons in the atomic nucleus. The IBM-1 Hamiltonian structure takes the form of [1, 2]:

$$\hat{H} = \varepsilon (n_d) + a_o (\hat{P} \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) + a_2 (\hat{Q} \cdot \hat{Q}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) + a_4 (\hat{T}_4 \cdot \hat{T}_4) \quad \dots\dots\dots (1)$$

Where  $\varepsilon$ ,  $a_o$ ,  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$  are the model parameters,  $P$ ,  $L$ ,  $Q$ ,  $T_3$  and  $T_4$  are the pairing, angular momentum, quadrupole, octopole and hexadecapole operators, respectively.  $n_d$  is the d-boson number operator, and all operators in the Hamiltonian structure are as follows [1-4]:

$$\hat{P} = \frac{1}{2} [(\tilde{d} \cdot \tilde{d}) - (\tilde{s} \cdot \tilde{s})] = \frac{1}{2} (\tilde{d}^2 - \tilde{s}^2)$$

$$T_l = [d^\dagger \otimes \tilde{d}]^l \quad l=0,1,2,3,4,\dots$$

$$\hat{L} = \sqrt{10} [d^\dagger \otimes \tilde{d}]^1 = \sqrt{10} \hat{T}_1,$$

$$\hat{Q} = [d^\dagger \otimes \tilde{s} + s^\dagger \otimes \tilde{d}]^2 - \frac{\sqrt{7}}{2} [d^\dagger \otimes \tilde{d}]^2 = [d^\dagger \otimes \tilde{s} + s^\dagger \otimes \tilde{d}]^2 - \frac{\sqrt{7}}{2} T_2$$

$$\hat{T}_3 = [d^\dagger \otimes \tilde{d}]^3,$$

$$\hat{T}_4 = [d^\dagger \otimes \tilde{d}]^4,$$

$$\hat{n}_d = \sqrt{5} \hat{T}_0$$

The E2 operator is used to compute the B(E2) values. The quantity of bosons must be conserved because the E2 transition operator (electric quadrupole transition operator) must be a Hermitian tensor of rank two. The universal E2 operator can be written with these restrictions:

$$T(E2) = e_B [(d^\dagger \tilde{s} + s^\dagger \tilde{d}) + \chi(d^\dagger \tilde{d})^{(2)}] = e_B Q \quad \text{-----} \quad (2)$$

Here  $e_B$  represents the effective boson charge. The metric  $\chi$  specifies the relevance of the two terms. The E2 operator, which shares the same structure as the  $Q$  operator in the Hamiltonian structure, consists of a single component which alters  $n_d$  by unity and another that leaves  $n_d$  unaltered with the ratio of the two components determined by the  $\chi$  variables [5-9].

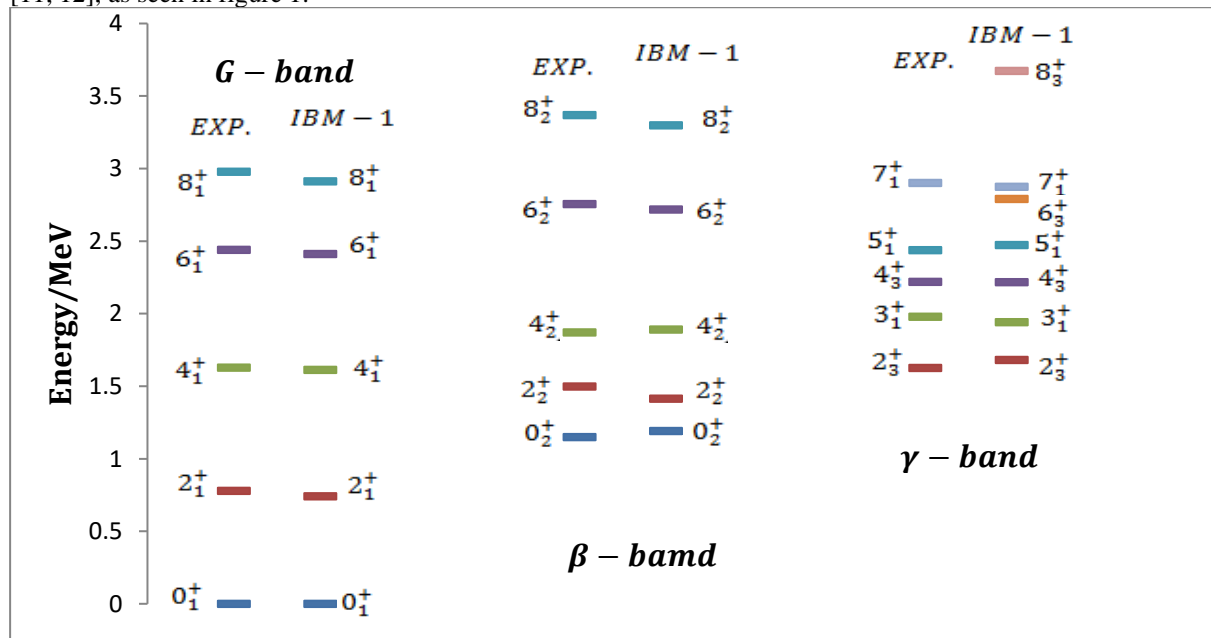
### 3. Results and Discussion

By using the parameters in tables (1) in the Hamiltonian equations of IBM-1, the result for low-lying positive parity energy spectra was obtained for  $^{96}_{42}\text{Mo}$  and  $^{98}_{42}\text{Mo}$ , respectively. These low-lying energy spectra obtained by IBM-1 were within the SU(5) limit for both isotopes. The IBM-1 Hamiltonian structure was constructed and solved in the SU(5) basis by using computer program code PHINT [10]. The parameters were free that were determined to almost exactly reproduce the excitation-energy of all positive parity levels.

**Table 1:** The IBM-1 Hamiltonian parameters used in the present study for the IBM-1 calculation of  $^{98}_{42}\text{Mo}$  isotope

$^{98}_{42}\text{Mo}$						CHI	SO6
$\epsilon$ (MeV)	$a_0$ (MeV)	$a_1$ (MeV)	$a_2$ (MeV)	$a_3$ (MeV)	$a_4$ (MeV)		
0.4310	0.0150	0.0099	0.0230	0.0541	0.0112	0.000	1.0000
0.4480	0.0140	0.0000	0.0230	0.0541	0.0112	0.000	1.0000

The most accurate fit to the Hamiltonian variables Eq.(1) utilized in this inquiry is shown in table (1), producing an excellent match with the estimated energy levels in the project and the associated experimental data for  $^{96}_{42}\text{Mo}$  [11, 12], as seen in figure 1.



**Figure (1):** a comparison between the experimental and IBM-1 low-lying positive parity states of the ground, beta and gamma bands for  $^{96}_{42}\text{Mo}$

Table (1) also shows the best match for the Hamiltonian parameters Eq.(1) utilized in this investigation, giving the greatest alignment between the computed energy levels in this work and their associated experimental results for  $^{98}_{42}\text{Mo}$  [13, 14], as shown in figure 2.

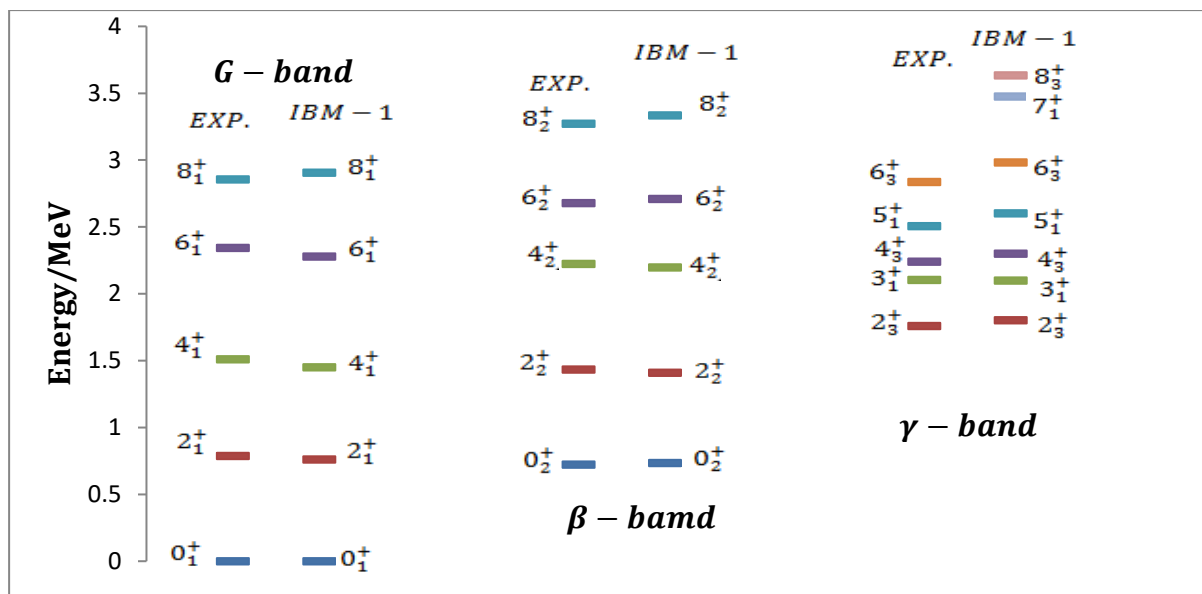


Figure (2): a comparison between the experimental and IBM-1 low-lying positive parity states of the ground, beta and gamma bands for  $^{98}_{42}\text{Mo}$

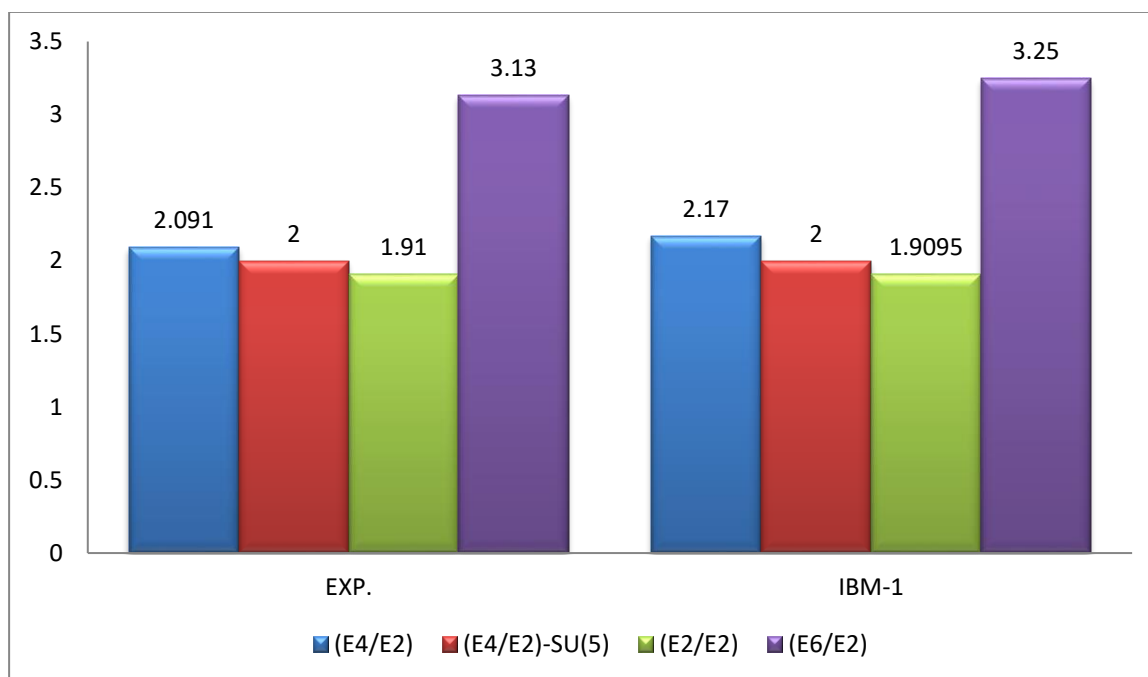
The boson E2 operator in IBM-1, equation (2), was used for calculating the E2 transition rates and the quadrupole moments for the low-lying excited states of the considered  $^{98}_{42}\text{Mo}$ . According to this concept, the value of the IBM-1's effective charge  $e_B$  was obtained by normalizing to  $B(E2: 2_1^+ \rightarrow 0_1^+)$  experimental data. The (IBMT-code)  $B(E2)$  values employed in the present investigation are shown in table 2.

Table 2: The experimental [11-16] and calculated value of  $B(E2)$  transition and quadrupole moments for  $^{96,98}_{42}\text{Mo}$ .

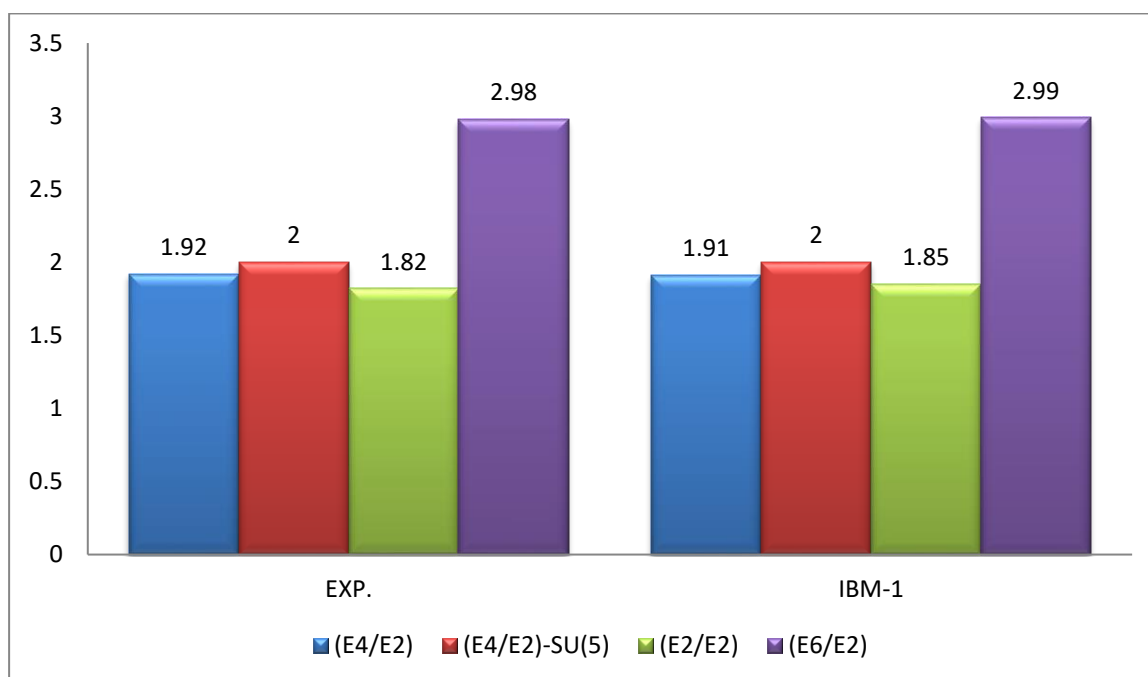
Parameters	$e_B = 0.12300 \text{ e. b}$	$\chi = -1.3500$	$SO(6) = 1.000$	$e_B = 0.12300 \text{ e. b}$	$\chi = -1.3500$	$SO(6) = 1.00$
Isotopes	$^{96}_{42}\text{Mo.}$		$^{98}_{42}\text{Mo.}$			
$B(E2) \text{ e}^2 \text{ b}^2 / Q(J) \text{ e. b}$	Exp.	IBM-1	Exp.	IBM-1		
$B(E2: 2_1^+ \rightarrow 0_1^+)$	0.05405(0.078%)	0.0563	0.05368(0.107%)	0.05610		
$B(E2: 2_1^+ \rightarrow 0_2^+)$		0.0249	0.05636(0.537%)	0.03640		
$B(E2: 2_2^+ \rightarrow 0_1^+)$	0.0031(0.026%)	0.0018	0.00011(0.01%)	0.00024		
$B(E2: 2_2^+ \rightarrow 0_2^+)$		0.0206	0.02147(1.879%)	0.02390		
$B(E2: 2_3^+ \rightarrow 0_1^+)$		0.0001	0.00258(0.019%)	0.00013		
$B(E2: 2_3^+ \rightarrow 0_2^+)$		0.0629	0.0644(0.215%)	0.07920		
$B(E2: 2_2^+ \rightarrow 2_1^+)$	0.0470(0.78%)	0.0667	0.01610(1.34%)	0.01100		
$B(E2: 2_3^+ \rightarrow 2_1^+)$		0.0007	0.11809(1.07%)	0.12310		
$B(E2: 2_3^+ \rightarrow 2_2^+)$		0.0334		0.04290		
$B(E2: 4_1^+ \rightarrow 2_1^+)$	0.10444(2.09%)	0.1097	0.12346(1.34%)	0.12310		
$B(E2: 4_1^+ \rightarrow 2_2^+)$		0.0086		0.01010		
$B(E2: 4_1^+ \rightarrow 2_3^+)$		0.0334		0.04290		
$B(E2: 4_2^+ \rightarrow 2_1^+)$	0.00496(0.157%)	0.0032		0.00310		
$B(E2: 4_2^+ \rightarrow 2_2^+)$	0.06005(1.83%)	0.0630		0.08300		
$B(E2: 4_2^+ \rightarrow 2_3^+)$		0.0222		0.02580		
$B(E2: 4_3^+ \rightarrow 2_3^+)$		0.0922		0.1261		
$B(E2: 4_2^+ \rightarrow 4_1^+)$		0.0573		0.07540		
$B(E2: 6_1^+ \rightarrow 4_1^+)$		0.1204		0.15840		
$B(E2: 8_1^+ \rightarrow 6_1^+)$		0.1264		0.17510		
$Q2_1^+$	$-0.20^{+0.08}_{-0.08}$	-0.3001	$-0.26^{+0.09}_{-0.09}$	-0.39740		

The results of this study for B(E2) transitions and quadrupole moment values are in a good agreement with the experimental values.

The energy ratios  $(\frac{E4_1^+}{E2_1^+})$ ,  $(\frac{E2_2^+}{E2_1^+})$ , and  $(\frac{E6_1^+}{E2_1^+})$  of the selected  $^{96,98}_{42}\text{Mo}$  isotopes were calculated in the frame work of IBM-1 together with their corresponding experimental values, as shown in figures (3 and 4) for  $^{96}_{42}\text{Mo}$  and  $^{98}_{42}\text{Mo}$ , respectively. The figures demonstrate that both isotopes of molybdenum evidence are considered a vibrational symmetry of SU(5) chain group since they are stable and have  $(\frac{E4_1^+}{E2_1^+})$  ratio values close to the SU(5) chain group. In general, the IBM-1 computations of the aforementioned ratio values agree with the experimental energy ratio values.

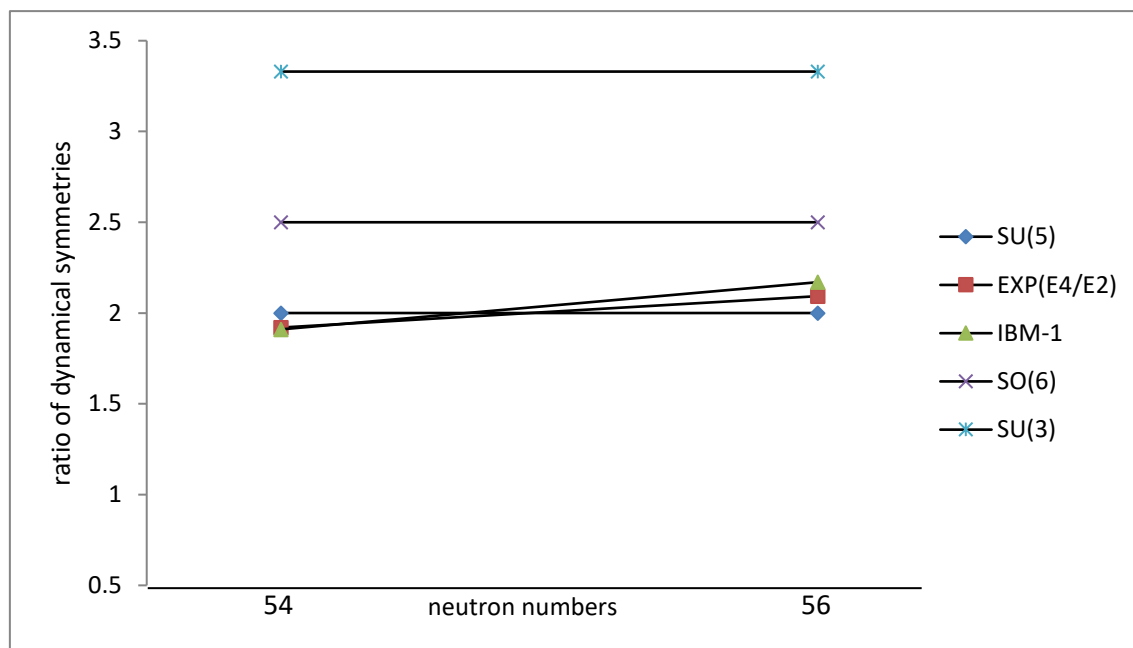


**Figure (3):** the experimental and calculated values of the energy ratios  $(\frac{E4_1^+}{E2_1^+})$ ,  $(\frac{E2_2^+}{E2_1^+})$ , and  $(\frac{E6_1^+}{E2_1^+})$  for  $^{96}_{42}\text{Mo}$ .



**Figure (4):** the experimental and calculated values of the energy ratios  $(\frac{E4_1^+}{E2_1^+})$ ,  $(\frac{E2_2^+}{E2_1^+})$ , and  $(\frac{E6_1^+}{E2_1^+})$  for  $^{98}_{42}\text{Mo}$ .

The dynamical symmetry properties according to IBM-1 can be investigated by knowing the ratio  $\frac{E_{4_1^+}}{E_{2_1^+}}$ . If the ratio is near to 2, the nuclei will behave like a vibrational of the group chain SU(5) or will be stable. Figure (5) depicts both stable molybdenum characteristics, such as vibrational nuclei in the SU(5) group chain.



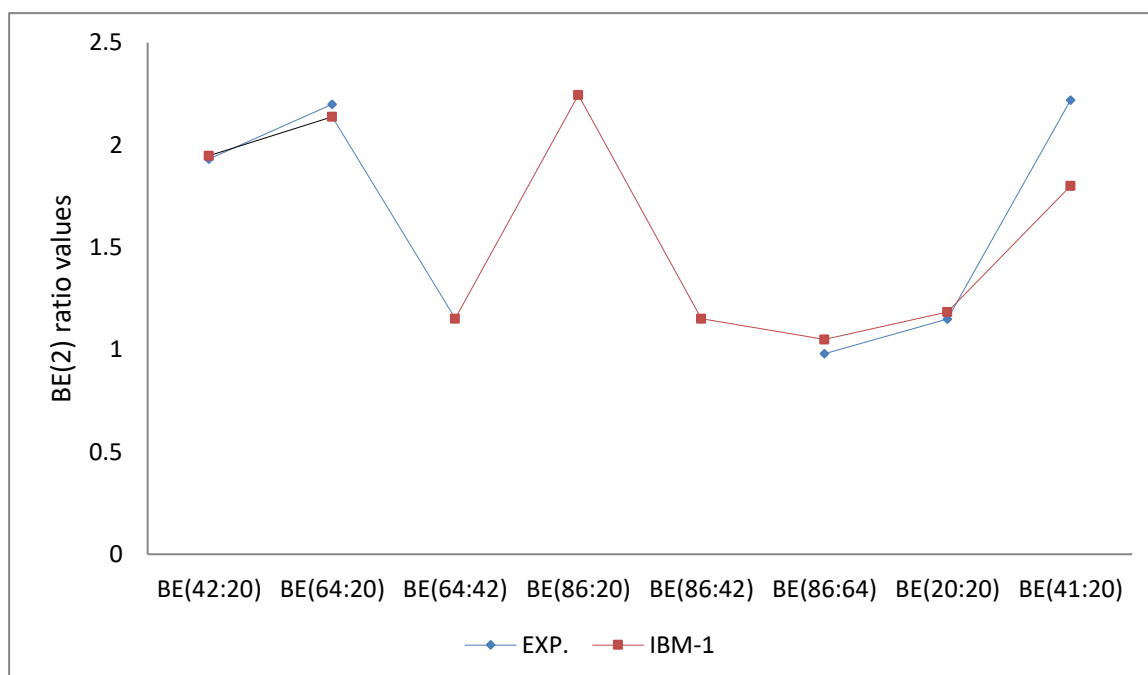
**Figure (5):** the experimental and calculated values of the energy ratios ( $\frac{E_{4_1^+}}{E_{2_1^+}}$ ) according to the SU(5) dynamical symmetry

The ratio  $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$  is a component of nuclei structure. As a result, the ratio obtained in the present work is the best fit agreement to the experimental data [16], as illustrated in table (3).

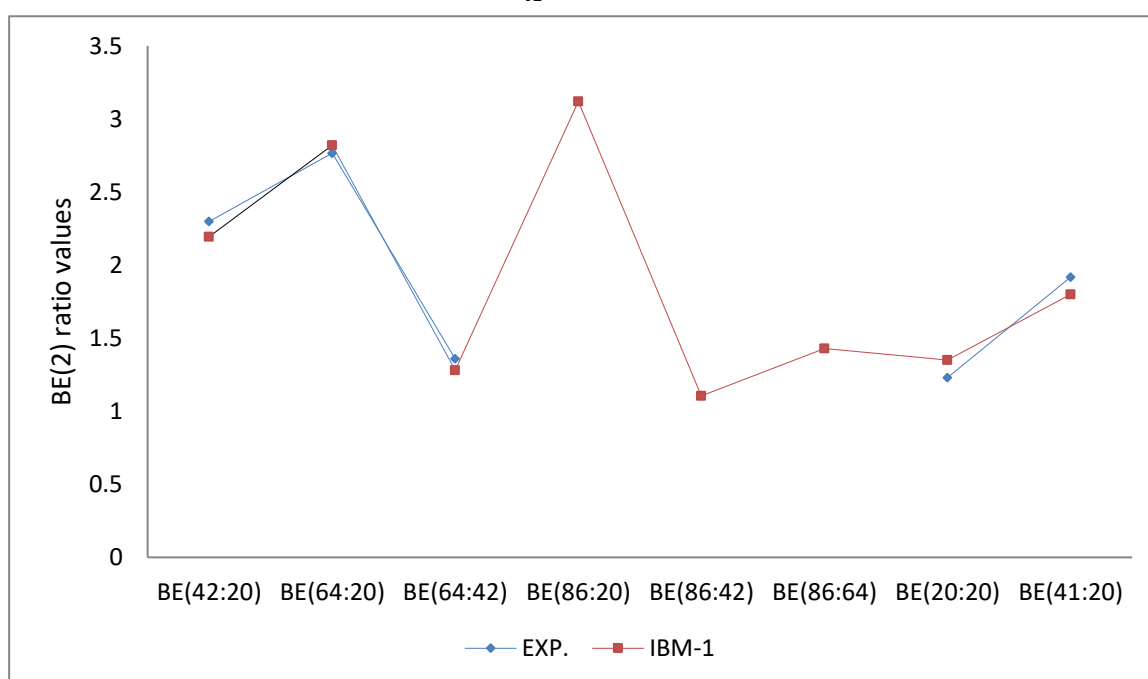
**Table 3:** Comparison between experimental and calculated values of the ratio  $B(E2: 4_1^+ \rightarrow 2_1^+)/B(E2: 2_1^+ \rightarrow 0_1^+)$  for  $^{96,98}_{42}\text{Mo}$ .

Isotopes	Exp. [12, 13]	IBM-1
$^{96}_{42}\text{Mo}$	1.932285	1.9484
$^{98}_{42}\text{Mo}$	2.299	2.1943

The good agreement between theoretical and experimental B(E2) values motivated extending the results to neutron-rich nuclei. The B(E2) transition ratios are useful tools for explaining nuclear structural features. Figures (6 and 7) compare between the observed and theoretical ratios of the B(E2) transition for both molybdenum isotopes [17].



**Figure 6:** the comparison between the experimental and IBM-1 ratios of  $B(E2: j_i^+ \rightarrow j_f^+)/B(E2: j_i^+ \rightarrow j_f^+)$  for  $^{96}_{42}\text{Mo}$  nuclide



**Figure 7:** the comparison between the experimental and IBM-1 ratios of  $B(E2: j_i^+ \rightarrow j_f^+)/B(E2: j_i^+ \rightarrow j_f^+)$  for  $^{98}_{42}\text{Mo}$  nuclide

## Conclusion

IBM-1 was used to calculate the theoretical values for the isotopes  $^{96,98}_{42}\text{Mo}$  with a proton number of 42. These isotopes had six and seven bosons, respectively, and were stable nuclei with SU(5) dynamical symmetry according to the present study. The energy levels of the low-lying positive parity states and the B(E2) values for these isotopes were obtained through using IBM-1, and then were compared with experimentally observed values. The results indicated a high degree of consistency, indicating that IBM-1 is capable of describing the energy levels of these isotopes. Furthermore, according to the current findings, additional investigation on " $^{96,98}_{42}\text{Mo}$  isotopes" B(E2) values will be required in order to identify the strength of E2 transitions.

## References

- [1]. Iachello, Francesco, F. Iachello, and P. Van Isacker. *The interacting boson-fermion model*. Cambridge University Press, 1991.
- [2]. Iachello, F., N. V. Zamfir, and R. F. Casten. "Phase coexistence in transitional nuclei and the interacting-boson model." *Physical review letters* 81.6 (1998): 1191.
- [3]. Hady, Heiyam Najy, and Ruqaya Talib Kadhim. "Mixed symmetry states in 96mo and 98ru isotones in the framework of interacting boson model." *3c Empresa: investigación y pensamiento crítico* 12.1 (2023): 243-255.
- [4]. Hu, Bao-Yue, et al. "Quadrupole and octupole states in 152Sm using the proton-neutron interacting boson model." *Journal of Physics G: Nuclear and Particle Physics* (2023).
- [5]. ghafoor, berun nasraddin, & Shwan, Y. H. (2023). Investigate shielding of standard materials and glass for stopping 662 KeV gamma ray penetrations. *Tikrit Journal of Pure Science*, 28(2), 83–87. <https://doi.org/10.25130/tjps.v28i2.1274>
- [6]. Sabbar, Ali N., and Saad N. Abood. "Investigation of nuclear structure for 130-136NCe nuclei within IBM-1 and DDM models framework." *AIP Conference Proceedings*. Vol. 2457. No. 1. AIP Publishing LLC, 2023.
- [7]. El-Gendy, H. S. "Nuclear structure of even-even Ytterbium isotopes 166– 176Yb." *Nuclear Physics A* 1001 (2020): 121883.
- [8]. Sulaiman, Huda Talal, et al. "Cd, sn, te, and xe isobars at (A= 116) under the framework of (ibm-1)." *NeuroQuantology* 18.1 (2020): 99.
- [9]. El-Gendy, H. S. "Theoretical study of nuclear collective phenomena and broken SU (3) symmetry of even-even 238– 244Pu isotopes." *Nuclear Physics A* 1006 (2021): 122117.
- [10]. Scholten, Olaf. "The program package PHINT." (1979).
- [11]. Abriola, D., & Sonzogni, A. A. (2008). Nuclear Data Sheets for A = 96. *Nuclear Data Sheets*, 109(11), 2501–2655.
- [12]. Thomas, T., et al. "Nuclear structure of <sup>96,98</sup>Mo: Shape coexistence and mixed-symmetry states." *Nuclear Physics A* 947 (2016): 203-233.
- [13]. Jun Chen, Balraj Singh "Nucl. Data Sheets A=98" Vol 164, 1 (2020)
- [14]. Zielńska, M., et al. "Electromagnetic structure of <sup>98</sup>Mo." *Nuclear Physics A* 712.1-2 (2002): 3-13.
- [15]. Devi, Vidya. "Structure and odd--even staggering of Mo, Ru, and Pd even--even nuclei in the framework of IBM-1." *Turkish Journal of Physics* 37.3 (2013): 330-347.
- [16]. Paradis, P., et al. "Static quadrupole moment of the first 2+ state in Mo 98." *Physical Review C* 20.3 (1979): 1201.
- [17]. Jin-Fu, Zhang, et al. "Mixed Symmetry States in Even-Even <sup>96-108</sup>Mo Nuclei." *Communications in Theoretical Physics* 37.3 (2002): 335.