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Class Nearly kahler Manifold of W – Projective Curvature Tensor

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ABSTRACT
The current study deals with new three classes of the nk ("nearly kahler") manifold of w – projective curvature tensor. The aim of this paper to calculate differential - geometrical and topological properties closest for new classes $\overline{w}_1, \overline{w}_2$, and $\overline{w}_3$, through it ,an equivalence relationship was obtained between these classes and one of or more the tensor compounds and the components of curvature tensor and with adjoint G-structure space. Finally, we discover a relationship between w_1,w_2 , w_3 with each other.

Introduction
The concept of AH-Almost Hermitian structures states that there is a general rule for classifying AH-structures using the second order symmetry features of the Riemann-Christoffel tensor's invariants of differential geometry. Based on the theory advanced by A. Gray and developed within a number of respective works [1] and [2]the important to understanding the differential-geometrical characteristics of Kahler manifolds is to establish the identities of them that satisfies. Gray and Hervella[3] founded that the action of the unitary group U(n) on the space of all tensors of type (3,0) decomposed this space in to sixteen classes. Following are the components that make up the Riemann curvature tensor:

$\overline{W}_1$ if $< W(\alpha,ß)\theta,\gamma >=< W(\alpha,ß)Q\theta,Q\gamma >$;

$\overline{W}_2$ if $< W(\alpha,ß)\theta,\gamma > = < W(Q\alpha,Qß)\theta,\gamma >+< W(Q\alpha,ß)Q\theta,\gamma >+< W(Q\alpha,ß)\theta,Q\gamma >$;

$\overline{W}_3$ if $< W(\alpha,ß)\theta,\gamma >=< W(Q\alpha,Qß)Q\theta,Q\gamma >$.

The AH-structures belonging to the class W_i have a tensor R that fulfills the identity W_i. If AH-any subclass of H-structures is named $\cap W_i$ =0, where i is 1, 2, or 3,

In this paper, we will generalize these relationships, definitions and theories related to them for NK – Nearly Kahler manifold of W – projective curvature tensor. In 2018 Ali A. Shihab and Dhabia'a M. Ali where studied classes of almost Hermitian manifold [4]. In the study also concentrates generalized conharmonic curvature tensor of Vaisman -Gray manifold.

Preliminaries

Assume that M is a smooth manifold of dimension-2n; $C^\infty(M)$ is algebra of smooth functions on M; $\alpha(M)$ the module of smooth vector fields on M;and that $g =<.,.> -$ Riemannian metrics; $\tilde{N}$ – Riemannian connection of the metrics g on M; d - the operator of exterior differentiation. Additional all manifold, Tensor fields, and other objects are assumed to be of class C^∞.So Almost Hermition (is shorter, AH) structure on a manifold M the pair (Q, g) ,where Q-almost complex structure ($Q^2 = id$) on M, g =<.,.> −(pseudo) Riemannian metric on M .In this case$< Q\alpha, Qß > = < \alpha, ß >$; $\alpha, ß \in \alpha(M)$.

Definition 1 [5]
The manifold (W, Q, g) denotes to as manifold of a class:
1)$\overline{W}_1$ if $< W(\alpha, ß)\theta, \gamma >=< W(\alpha, ß)Q\theta, Q\gamma >$;
2)$\overline{W}_2$ if $< W(\alpha, ß)\theta, \gamma > = < W(Q\alpha, Qß)\theta, \gamma > + < W(Q\alpha, ß)Q\theta, \gamma > + < W(Q\alpha, ß)\theta, Q\gamma >$;
3) $\overline{W}_3$ if $< W(\alpha, ß)\theta, \gamma >=< W(Q\alpha, Qß)Q\theta, Q\gamma >$.
Note 2
NK – manifold of class
$W_0 = W_3 = W_5 = W_6$
Which are also manifold of a class $\overline{W}_3$.
It is most clear when the curvature identities are expressed in terms of a spectrum. Generalized projective curvature tensor.
Theorem 3
Assume that Z $= (Q, g =<.,.>)$ is NK ("Nearly Kahler") structure. Consequently, the following propositions are equivalent:
(1) Z- Structure of a class $\overline{W}_3$;
(2) $W_{(0)} = 0$; and
(3)The identities $W^{a}_{bcd} = 0$ on space of the adjoint G-structure are acceptable.
proof.
Assume that Z – structure of a class $\overline{W}_3$. Clearly, it is equal to identity $W(\alpha, ß)\theta + QW(Q\alpha, Qß)Q\theta = 0$; $\alpha, ß, \theta \in \alpha(M)$.
By definition of a spectrum tensor
$W(\alpha, ß)\theta = W_0(\alpha, ß)\theta + W_1(\alpha, ß)\theta + W_2(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_4(\alpha, ß)\theta + W_5(\alpha, ß)\theta + W_6(\alpha, ß)\theta + W_7(\alpha, ß)\theta; \alpha, ß, \theta\ \alpha(M)$.
$Q\circ W(Q\alpha, Qß)Q\theta = Q\circ W_0(Q\alpha, Qß)Q\theta + Q\circ W_1(Q\alpha, Qß)Q\theta + Q\circ W_2(Q\alpha, Qß)Q\theta + Q\circ W_3(Q\alpha, Qß)Q\theta + Q\circ W_4(Q\alpha, Qß)Q\theta + Q\circ W_5(Q\alpha, Qß)Q\theta + Q\circ W_6(Q\alpha, Qß)Q\theta + Q\circ W_7(Q\alpha, Qß)Q\theta = W(\alpha, ß)\theta = W_0(\alpha, ß)\theta - W_1(\alpha, ß)\theta - W_2(\alpha, ß)\theta - W_3(\alpha, ß)\theta - W_4(\alpha, ß)\theta - W_5(\alpha, ß)\theta - W_6(\alpha, ß)\theta - W_7(\alpha, ß)\theta; \alpha, ß, \theta\ \alpha(M)$.
Putting term by these identities, will be received:
$W(\alpha, ß)\theta + QW(Q\alpha, Qß)Q\theta = \{W_0(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_5(\alpha, ß)\theta + W_6(\alpha, ß)\theta\}$.
With means, the identity $W(\alpha, ß)\theta + QW(Q\alpha, Qß)Q\theta = 0$ is equivalent to that $W_0(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_5(\alpha, ß)\theta + W_0(\alpha, ß)\theta$ and this identity is equivalent to identities $W_0 = W_3 = W_5 = W_6 = 0$.
the established characteristics of the adjoint G-space structure's are equal to relations:
$W^{a}_{\hat{b}cd} = W^{a}_{\hat{b}\hat{c}\hat{d}} = W^{a}_{\hat{b}c\hat{d}} = 0$.
By virtue of materiality tensor W received relations which are same to relations
$W^{a}_{bcd} = 0$, i.e. identity $W_0(\alpha, ß)\theta = 0$.
Theorem 4
Assume that $Z = (Q, g =<\cdot,\cdot>)$ be NK ("Nearly Kahler") structure, Consequently, the following propositions are equivalent
(1) Z- Structure of a class $\overline{W}_2$;
(2) $W_0 = W_7 = 0$; and
(3) The identities $W^{a}_{bcd} = W^{a}_{\hat{b}\hat{c}\hat{d}} = 0$ on space of the attached G-structure are acceptable.

Proof:
Assume that Z- structure of a class $\overline{W}_2$. We shall duplicate identity $\overline{W}_2$ in with everyone in place, this identity will be computed using the notion of a spectrum tensor as follows:
$W(\alpha, ß)\theta = W_0(\alpha, ß)\theta + W_1(\alpha, ß)\theta + W_2(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_4(\alpha, ß)\theta + W_5(\alpha, ß)\theta + W_6(\alpha, ß)\theta + W_7(\alpha, ß)\theta$;
1) $W(Q\alpha, Qß)\theta = W_0(Q\alpha, Qß)\theta + W_1(Q\alpha, Qß)\theta + W_2(Q\alpha, Qß)\theta + W_3(Q\alpha, Qß)\theta + W_4(Q\alpha, Qß)\theta + W_5(Q\alpha, Qß)\theta + W_6(Q\alpha, Qß)\theta + W_7(Q\alpha, Qß)\theta = -W_0(\alpha, ß)\theta + W_1(\alpha, ß)\theta + W_2(\alpha, ß)\theta - W_3(\alpha, ß)\theta - W_4(\alpha, ß)\theta + W_5(\alpha, ß)\theta + W_6(\alpha, ß)\theta - W_7(\alpha, ß)\theta$;
2)$W(Q\alpha, ß)Q\theta = W_0(Q\alpha, ß)Q\theta + W_1(Q\alpha, ß)Q\theta + W_2(Q\alpha, ß)Q\theta + W_3(Q\alpha, ß)Q\theta + W_4(Q\alpha, ß)Q\theta + W_5(Q\alpha, ß)Q\theta + W_6(Q\alpha, ß)Q\theta + W_7(Q\alpha, ß)Q\theta; = -W_0(\alpha, ß)\theta - W_1(\alpha, ß)\theta + W_2(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_4(\alpha, ß)\theta + W_5(\alpha, ß)\theta - W_6(\alpha, ß)\theta - W_7(\alpha, ß)\theta$.
3)$QW(Q\alpha, ß)\theta = QW_0(Q\alpha, ß)\theta + QW_1(Q\alpha, ß)\theta + QW_2(Q\alpha, ß)\theta + QW_3(Q\alpha, ß)\theta + QW_4(Q\alpha, ß)\theta + QW_5(Q\alpha, ß)\theta + QW_6(Q\alpha, ß)\theta + QW_7(Q\alpha, ß)\theta; = -W_0(\alpha, ß)\theta - W_1(\alpha, ß)\theta + W_2(\alpha, ß)\theta + W_3(\alpha, ß)\theta - W_4(\alpha, ß)\theta - W_5(\alpha, ß)\theta + W_6(\alpha, ß)\theta + W_7(\alpha, ß)\theta$.
Substituting these breakdown in the prior equality, we will obtain:
$W(\alpha, ß)\theta - W(Q\alpha, Qß)\theta - W(Q\alpha, ß)Q\theta + QW(Q\alpha, ß)\theta + QW(Q\alpha, ß)\theta = 2\{ W_0(\alpha, ß)\theta + W_3(\alpha, ß)\theta + W_6(\alpha, ß)\theta + W_7(\alpha, ß)\theta \}$
This identity is equivalent to that
$W_0(\alpha, ß)\theta = W_3(\alpha, ß)\theta = W_5(\alpha, ß)\theta = W_7(\alpha, ß)\theta = 0$.
Additionally, these identities on the space of the adjoint G-structure are comparable to identities.
$W^{a}_{bcd} = W^{a}_{\hat{b}\hat{c}\hat{d}} = W^{a}_{\hat{b}cd} = W^{a}_{\hat{b}c\hat{d}} = W^{a}_{\hat{b}\hat{c}\hat{d}}$.
The received relations are identical to these by reason of the materiality tensor W: $W^{a}_{bcd} = W^{a}_{\hat{b}\hat{c}\hat{d}}$, i.e. to identities $W_0(\alpha, ß)\theta = W_0(\alpha, ß)\theta$.
Back, let for NK- manifold identities $W_0(\alpha, ß)\theta = W_7(\alpha, ß)\theta = 0$ are executed.
Then we have:
$W(\alpha, ß)\theta - W(\alpha, Qß)Q\theta - W(Q\alpha, ß)Q\theta - W(Q\alpha, ß)Q\theta = 0$
i.e.
$W(\alpha, ß)\theta = W(Q\alpha, ß)Q\theta = W(Q\alpha, Qß)\theta = W(\alpha, Qß)\theta$
In the received identity instead of $W(\alpha, Qß)\theta$ we shall put the value $ß \to Qß$ and $\alpha \to Q\alpha$, i.e $W(\alpha, Qß)Q\theta = -QW(Q\alpha, ß)\theta$.
Then
$W(\alpha, ß)\theta = W(Q\alpha, Qß)\theta + W(Q\alpha, ß)Q\theta - QW(Q\alpha, Qß)\theta$
i.e.
As a result, the manifold fulfills the identity.$\overline{W}_2$.
It is also proven by the next theorem.
Theorem 5

Assume that $Z = (Q, g = <.,.>)$ be NK ('' Nearly Kahler '')structure. Consequently, the following propositions are equivalent to:

(1) θ –structure of a class $\bar{W}_1$;

(2) $W_0 = W_4 = W_7 = 0$;

(3) The identities $W^a_{bcd} = W^a_{\hat{b}cd} = W^a_{\hat{b}\hat{c}\hat{d}}$ on space of the attached G-structure are acceptable.

Proof :

Assume that Z is a structure of a class $\bar{W}_1$. Clearly, it is comparable to identity

$< W(\alpha,ß)\theta, \gamma > = < W(\alpha,ß)Q\theta, Q\gamma >$

and we get $W(\alpha,ß)\theta + QW(\alpha,ß)Q\theta = 0$; $\alpha, ß, \theta \in \alpha(M)$.

By definition of a spectrum tensor

1) $W(\alpha,ß)\theta = W_0(\alpha,ß)\theta + W_1(\alpha,ß)\theta + W_2(\alpha,ß)\theta + W_3(\alpha,ß)\theta + W_4(\alpha,ß)\theta + W_5(\alpha,ß)\theta + W_6(\alpha,ß)\theta + W_7(\alpha,ß)\theta$; $\alpha,ß,\theta \in \alpha(M)$.

2) $Q_oW(\alpha,ß)Q\theta = Q_o W_0(\alpha,ß)Q\theta + Q_oW_1(\alpha,ß)Q\theta + Q_oW_2(\alpha,ß)Q\theta + Q_oW_3(\alpha,ß)Q\theta + Q_oW_4(\alpha,ß)Q\theta + Q_oW_5(\alpha,ß)Q\theta + Q_oW_6(\alpha,ß)Q\theta + Q_o W_7(\alpha,ß)Q\theta$; $= -W_0(\alpha,ß)\theta - W_1(\alpha,ß)\theta - W_2(\alpha,ß)\theta - W_3(\alpha,ß)\theta + W_4(\alpha,ß)\theta - W_5(\alpha,ß)\theta - W_6(\alpha,ß)\theta + W_7(\alpha,ß)\theta$; $\alpha, ß, \theta \in \alpha(M)$.Putting (1) and (2) in $W(\alpha,ß)\theta + QW(\alpha,ß)Q\theta$ means, this identity is equivalent to that

$W_{(0)}(\alpha,ß)\theta + W_4(\alpha,ß)\theta\ W_0(\alpha,ß)\theta = 0$.

And this identity is equivalent to identities$W_{(0)} = W_4 = W_7 = 0$.

The established identities in space of the adjoint G-structure are equal to relations:

$W^a_{bcd} = W^a_{\hat{b}cd} = W^a_{\hat{b}\hat{c}\hat{d}} = 0$.

Corollary 6

Let $Z = (Q, g = <.,.>)$ be NK ('' Nearly Kahler '') structure.Afterward, the inclusions listed below of classes $\bar{W}_1 \subset \bar{W}_2 \subset \bar{W}_3$ are acceptable.

proof:

Let Z - structure of a class $\overline{(W)}_1$. Obviously, it is equivalent to $(W)_0 = (W)_4 = (W)_7 = 0$, By theorem 5 .

So (By theorem 4) $(W)_0 = (W)_7 = 0$, is equivalent to class $\overline{(W)}_2$.

Then $\overline{(W)}_1 \subset \overline{(W)}_2$.

Also the class $\overline{(W)}_3$ is equivalent to $(W)_0 = 0$, that is clear from theorem 5.

Thus $\overline{(W)}_1 \subset \overline{(W)}_2 \subset \overline{(W)}_3$.

Conclusion

Find new classes $\bar{W}_0(N.K), \bar{W}_1(N.K) and\ \bar{W}_3(N.K)$ and proved the structure. $\bar{W}_3(N.K)$is $\bar{W}_7(N.K) = 0$, and on space of the adjoint G-structure identities $W(N.K)^a_{\hat{b}\hat{c}\hat{d}} = 0$ are fair.

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كلاسات تنسر الاسقاط من النوع W في منطوي كوهلر التقريبي

علي خلف علي, علي عبد المجيد شهاب

قسم الرياضيات ، كلية التربية للعلوم الصرفة , جامعة تكريت ، تكريت ، العراق

الملخص

في هذا البحث ندرس كلاسات جديدة للتنزر الاسقاطي من نوع W لمنطوي كوهلر التقريبي مع نظريات جديدة اي اننا نجد فئات جديدة $\bar{W}_1, \bar{W}_2$ and $\bar{W}_3$, ثم قمنا بانشاء علاقة بينها وبين مكونات تنزر الانحناء الاسقاطي واخيرا نكتشف علاقة بين W_1, W_2 , W_3 مع بعضها البعض.