Tikrit Journal of Pure Science (2023) 28 (5): 65-73
Doi: https://doi.org/10.25130/tjps.v28i5.1389

TJPS

IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science
ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

Quantum Teleportation and Entanglement of the 5-Qubit State Over a GHZ-Like Channel

[1,a]Aseel H. Abdullah, [1,b]Shareef Faiq Sultan Al-Tikrit, [2]Abdullah H. Mohammed
[1]*Tikrit University - College of Science - Department of Physics - Salah al-Din - Iraq*
[2]*Kirkuk University - College of Education - Department of Physics - Kirkuk - Iraq*

Keywords: 5-Qubit, quantum circuit, quantum teleportation, entanglement, GHZ state.

ARTICLE INFO.

Article history:

-Received:	05 June 2023
-Received in revised form:	06 July 2023
-Accepted:	15 July 2023
-Final Proofreading:	24 Oct. 2023
-Available online:	25 Oct. 2023

Corresponding Author*:
Aseel H. Abdullah
Aseelhamd50@gmail.com

ABSTRACT

Quantum information processing has a wide range of uses, including the solution of complex algorithms, cryptography, dense coding, quantum computation, and quantum teleportation. Quantum entanglement, a phenomenon that is crucial to many different applications of quantum computation, is an essential resource for quantum computation, and the need for quantum circuits that generate entangled states is one of the ongoing needs for constructing quantum computers. In this study, we present a general approach to the design of quantum circuits that produce entangled states. The approach can be applied to the design of various entanglement circuits with any number of qubits (n-qubit systems), and it makes use of a set of CNOT and Hadamard gates, where the number of CNOT gates should always be (n-1) and each gate connects the two adjacent qubits. In this paper, a three-entangled teleportation scheme of a GHZ-like state (named after its inventor Greenberger-Horne-Zeilinger) through three particles as a quantum channel is presented. The probability of successful teleportation depends on the degree of entanglement of GHZ-like states. A 5-qubit quantum teleportation over a GHZ-like channel has also been used. And single-qubit gates are defined, which are Pauli gates. These gates are represented by arrays I, X, Y, and Z. The results in this paper are good and promising theoretical results in the field of quantum entanglement and quantum teleportation using the Mathematica program.

TJPS

Tikrit Journal of Pure Science (2023) 28 (5): 65-73
Doi: https://doi.org/10.25130/tjps.v28i5.1389

النقل الكمي والتشابك في حالة ٥–كيوبت على قناة تشبه GHZ

أسيل حمد عبدالله[1]، شريف فايق سلطان التكريتي[1]، عبدالله حمود محمد[2]

[1] جامعة تكريت – كلية العلوم – قسم الفيزياء – صلاح الدين – العراق

[2] جامعة كركوك – كلية التربية – قسم الفيزياء – كركوك – العراق

الملخص

معالجة المعلومات الكمومية لها مجموعة واسعة من الاستخدامات بما في ذلك حل الخوارزميات المعقدة والتشفير والترميز الكثيف والحساب الكمي، وهي ظاهرة مهمة للعديد من التطبيقات المختلفة للحساب الكمومي، مورداً أساسياً للحساب الكمي، والحاجة الى الدوائر الكمية التي تولد حالات متشابكة هي واحدة من الاحتياجات المستمرة لبناء اجهزة الكمبيوتر الكمومية. في هذه الدراسة نقدم مقاربة عامة لتصميم الدوائر الكمومية التي تنتج حالات متشابكة. يمكن تطبيق هذا النهج على تصميم دوائر التشابك المختلفة بأي عدد من الكيوبتات انظمة (ن-كيوبت) ويستخدم مجموعة من بوابات سي نوت وهادمارد حيث يجب أن يكون عدد بوابات سي نوت دائما (ن-١) وتربط كل بوابة بين الكيوبتين المتجاورين. في هذا البحث مخطط انتقال عن بعد ثلاثي متشابك لحالة تشبه GHZ (سميت على اسم مخترعها Greenberger-Horne-Zeilinger) من خلال ثلاث جسيمات كقناة كمية يعتمد احتمال النقل الأني الناجح على درجة تشابك الحالات الشبيهة بـ GHZ. كما تم استخدام النقل الأني الكمي ٥-كيوبت عبر قناة تشبه GHZ. ويتم تحديد بوابات احادية الكيوبت، وهي بوابات باولي. يتم تمثيل هذه البوابات بواسطة المصفوفات (I, X, Y, Z). تعتبر النتائج في هذه الورقة جيدة ونتائج نظرية واعدة في مجال التشابك الكمي والانتقال الأني الكمي باستخدام برنامج ماثمتكا.

الكلمات المفتاحية: ٥-كيوبت، دائرة كمية، النقل الأني الكمي، التشابك، حالة GHZ.

Introduction

Teleportation is a quantum task and can only be achieved by using shared entanglement states as a quantum channel [1]. The basic idea of the original quantum transmission scheme was the teleportation of an unknown single-qubit state from Alice (the sender) to Bob (the receiver) $|\psi> = \alpha|0> + \beta|1>$. By using 2 bits of classical communication using a shared entanglement state (Bell state). The most unconventional interpretation of quantum physics is entanglement [2]. In addition, it is a vital component in processing quantum information such as incredibly complex situations, Bell state, GHZ states, and cluster states [3]. As well as, it has been used in quantum teleportation, super-dense coding, one-way quantum computing, and several quantum algorithms. Entangled states are therefore a crucial step in proving their quantum nature. Quantum entanglement is becoming more significant in the field of quantum information science. Quantum information processing techniques including quantum teleportation [3,4] and others are said to depend on entanglement as a fundament resource. With the aid of some classical information, quantum teleportation can transmit an unknowable quantum state from a sender to a receiver at a different location via a quantum channel, as first proposed by Bennett et al. [1] and experimentally demonstrated by Khatri and Wilde [5]. Additionally, distributed entangled states enable the transmission of an unknown state across a great distance. The state of entanglement of three qubits can also be classified as GHZ-like. These computers require some technical considerations to be used effectively. It is from these considerations that the portals are created. The first set of gates is the Hadamard gate H, followed by the X, Y, and Z gates (these are the usual Pauli matrices), with just these gates, unitary transformation must be performed. Due to experimental limitations, not all qubits are coupled to one another, which is another technical detail [6].

Quantum Bit Theory

A quantum bit (qubit) is the primary building component of a quantum computer, unlike a bit or binary number in a conventional Earth computer, which can only have a value of 0 or 1. Qubits may be in a randomly stacked combination of the following statistics: $|\psi> = \alpha|0> + \beta|1>$, where ($\alpha$) and ($\beta$) are two random complicated numbers that have to adhere to the following requirement: $|\alpha|^2+|\beta|^2=1$. The effectiveness of quantum computing is due to these super-positions [7]. The most straightforward two-dimensional quantum mechanical system space is known as a qubit [8]. There are numerous words associated with qubits, which require that a single qubit be operated on by a set of four standard operators, known as Pauli factors, as specified by the matrices below:

$$I \equiv \sigma_0 \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ X \equiv \sigma_1 \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Y \equiv \sigma_2 \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix};\ Z \equiv \sigma_3 \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Since These matrices are constructed using an arithmetic foundation $|0>, |1>$. The character is usually escaped, and instead for Pauli operators type I, X, Y, or Z. The area of the operator vector per qubit is given as the basis of the Pauli operator, as mentioned above [9].

Atomic Gates

Qubits can be transformed using quantum gates like how classical qubits may be transformed using classical gates. Based on the principles of quantum physics and the quantum system's time development, this depends. The unitary operator describes this in detail. The quantum gate, however, operates on any arbitrary multi-qubit state since each quantum gate corresponds to a single operator U.$|\varphi>$ as in $|\varphi> \rightarrow\ |\varphi_{out}> = U|\varphi_{in}>$. From the output, the input state can be created by $|\varphi_{in}> = U^{\dagger}|\varphi_{out}>$, the quantum gate is always reversible as a result [10] .

Mono-Qubit Gate

A single qubit is represented as a vector using the notation $\binom{\alpha}{\beta}$ The single-qubit gate is a unary operator U that executes a mono transformation, and it is portrayed as a 2*2 matrix.

$|\psi>_{inp}$

$|\psi>_{out} = U|\psi>_{inp}$ (a)

$|\psi>_{out}$

$|\psi>_{inp} = U^{-1}|\psi>_{out}$ (b)

Figure (1): $a -$ Displays the basic design of a single-qubit random U gate, $b -$the opposite procedure is U^{-1} [11].

Any additional one-qubit state, as well as the qubit state $|\varphi>$ input, is defined as $|\varphi>$ out = U$|\varphi>$ input. However, because U^{-1} must exist for U to exist, the opposite procedure is $U^{-1}|\varphi>$ out = $|\varphi>$ input. The first figure single gates are introduced in this section and are as follows:

- X-Pauli gate or NOT gate: The gate of the unary operator X's matrix representation is:

$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. For the following input and output values, the X gate behaves as follows:
X|0> = |1> , X|1> = |0>, X (α|0> + β|1>) = (α|1> + β|0>).

- Z-Pauli gate: The following matrix format displays the Z-gate:

$Z = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. A Z-gate/ changes the single-qubit state in,

Z|0> = |0> , Z|1> = -|1 > , Z(α|0> + β|1>) = (α|0>-β |1>) .

- Y-Pauli gate: The Y gate, or σ_y as was previously indicated, is represented by the following matrix: Y= $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, the transformation of single-qubit states to:
Y|0> = |1> , Y|1> = |0>, Y(α|0> + β|1>) = (α|1> - β|0>).
- The Hadamard gate, also known as the H gate and a one-qubit gate, is represented as an array: H = $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, There are recognized Hadamard transforms.
- Phase gate: The following matrix also serves as a representation for this gate:

$P = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$, Two values will be used since it can have an endless number of values, and either $\theta = \frac{\pi}{2}$, consequently, get the S-gate shown in the figure (1).

$S = P(\frac{\pi}{2}) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$ The following diagram represents the S-states:
gate's S|0> =|0> , S|1> = i|1>, Or if $\theta = \frac{\pi}{4}$. The matrix's T-gate is obtained as follows:

$T = P(\frac{\pi}{4}) = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{2}}(1+i) \end{pmatrix}$. The P-gate is not often inverted, as should be noted [11].

Teleportation via GHZ-like State

Present the GHZ-like example $|\emptyset_G\rangle = 1/2$ (|001> + |010> |100> +|111>) as a reference, this example is identical to W-state (where W-state is one of the cases that also performs the teleportation process) and hence comparable to GHZ, it falls within the category of GHZ rather than W-state. But this situation can go as follows. Get two cases ready [12,13].

$|\varphi>_1 = |0>_1 + |1>_1$ and $|\varphi>_{23} = \frac{1}{\sqrt{2}}(|01> + |10>)_{23}$

They form a complex system as follows :

$|\emptyset_{G1}>_{123} = |\varphi>_1 \otimes |\varphi>_{23}$

$|\emptyset_{G1}>_{123} = (|0> + |1>)_1 \otimes \frac{1}{\sqrt{2}}(|01> + |10>)_{23}$

$= \frac{1}{2}(|001> + |010> + |110> + |101>)_{123}$ ……. (1)

Applying a non-quantum-controlled gate is $|\varphi_{G1}>_{123}$, that particle 2 is the target qubit and particle 1 is the control qubit.

$|\emptyset_G>_{123} = \frac{1}{\sqrt{2}}(|001> + |010> + |100> + |111>)_{123}$ ……..(2)

This GHz-like condition can already be created experimentally, and it will be practicable to create it. Assuming that the transmitter Alice desires teleportation.

$|\varphi>_1 = \alpha|0>_1 + \beta|1>_1$ …… (3)

As shown in the diagram, let's say two receivers are present, named Bob and Charlie. As a result, only one of the receivers may determine the latter's unknown status using the controlling end's measurement results; this recipient is referred to as the controlling party. Alice must pick one of the two to be the actual receiver, though. Here, the case will be considered in which Alice wants to automatically convey the unknown target state to someone. In exchange, Charlie is the one in control of this operation, as indicated in Figure (2) at Position 2. To ease the transfer, Alice employs the teleportation device [12,13]. The apparatus in this case has the following source of a three-particle entangled state:

$|\emptyset_G>_{234} = \frac{1}{2}(|001> + |010>+|100>+|111>)_{234}$………(4)

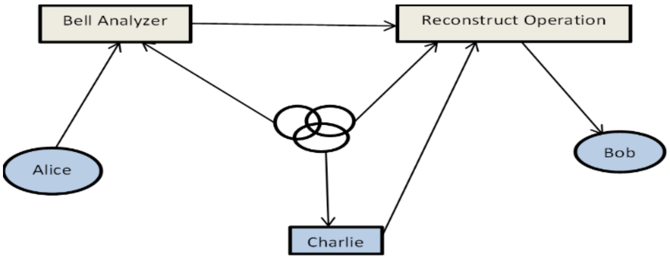

Figure (2): GHZ-like state to demonstrate the teleportation protocol, with Bob serving Charlie acting as the control terminal and the receiver [13].

One particle is retained by Alice during the procedure, while particles three and four are delivered to Bob and Charlie, respectively. It is stated as follows and represents the common product state, the target state, and the GHZ-like state:

$|\varphi>_1 \otimes |\emptyset_G>_{234} = (\alpha|0> + \beta|1>)_1 \otimes \frac{1}{2}(|001> + |010> + |100>+|111>)_{234}$

$= \frac{1}{2}(\alpha|000> + \beta|100> + \alpha|011> + \beta|111>)_{123} \otimes |1>_4 + \frac{1}{2}(\alpha|001> +\beta|101> +\alpha|010> + \beta|110>)_{123} \otimes |0>_4$ …….. (5)

The mentioned equation has been divided into two pieces so that the conversion can be presented clearly.

the first : $\frac{1}{2}[(\alpha|000> + \beta|100> + \alpha|011> + \beta|111>)_{123} \otimes |1>_4$

$= \frac{1}{4}(\alpha|000> + \alpha|110> + \beta|001> + \beta|111>)_{123} + (\alpha|000> - \alpha|110> - \beta|001> + \beta|111>)_{123} +(\alpha|011> + \alpha|101> + \beta|010> + \beta|100>)_{123} + (\alpha|011> - \alpha|101> - \beta|010> + \beta|100>)_{123}] \otimes |1>_4$

$= \frac{1}{2\sqrt{2}}[\frac{1}{\sqrt{2}}(|00> + |11>)_{12} \otimes (\alpha|0> + \beta|1>)_3 + \frac{1}{\sqrt{2}}(|00> - |11>)_{12} \otimes (\alpha|0> - \beta|1>)_3 + \frac{1}{\sqrt{2}}(|01> + |10>)_{12} \otimes (\alpha|0> + \beta|1>)_3 + \frac{1}{\sqrt{2}}(|01> - |10>)_{12} \otimes (\alpha|0> - \beta|1>)_3] \otimes |1>_4$ ………….. (6)

Section two : $\frac{1}{2}(\alpha|001> +\beta|101> +\alpha|010> + \beta|110>)_{123} \otimes |0>_4$

Tikrit Journal of Pure Science (2023) 28 (5): 65-73
Doi: https://doi.org/10.25130/tjps.v28i5.1389

$= \frac{1}{4}[\ (\beta|000> + \beta|110> + \alpha|001> + \alpha|111>)_{123} + (-\beta|000> + \beta|110> + \alpha|001> - \alpha|111>)_{123} + (\beta|011> + \beta|101> + \alpha|010> + \alpha|100>\ )_{123} + (-\beta|011> + \beta|101> + \alpha|010> - \alpha|100>)_{123}] \otimes |0>_4$

$= \frac{1}{2\sqrt{2}}\ [\ \frac{1}{\sqrt{2}}|00> + |11>)_{12} \otimes (\beta|0> + \alpha|1>)_3 + \frac{1}{\sqrt{2}}|00> - |11>)_{12} \otimes (-\beta|0> + \alpha|1>)_3 + \frac{1}{\sqrt{2}}\ |01> + |10>)_{12} \otimes (\beta|0> + \alpha|1>)_3 + \frac{1}{\sqrt{2}}|01> - |10>)_{12} \otimes (-\beta|0> + \alpha|1>)_3] \otimes |0>_4$ ……..(7)

With the use of Bell's state analyzers, Alice was able to measure the first and second particles. Alice then requested that Charlie measure particle 4 in the basis {|0>.|1>}, and they communicated to Bob via the conventional channel, accordingly, the findings of their measurements. Quantum teleportation does not, however, contradict the law of causation [12]. If Alice receives the result $|\emptyset^+>_{12}$, then Charlie's measurement result is $|1>_4$. Particle 3 transmitted to Bob collapses as a result.

$|\varphi>_3 = \alpha|0>_3 + \beta|1>_3$ ......(8)

The likelihood that Alice will experience four consequences, and Charlie will go through two outcomes is the target state $|\varphi>$ to which Alice wants to teleport to Bob as shown in equations (6) and (7). For Alice and Charlie, the measurement findings are broken down into eight separate sets. In some groups. Bob must carry out a straightforward one-step operation to recreate the necessary quantum state [12, 13].

Calculation and methods

The following circuit obtained using the Mathematica software is the one that generates entangled states similar to GHZ for five qubits, which means that the output state is an entangled state of four bases of equal capacitance. The circuit consists of two Hadamard gates, one operating on the first qubit and the second qubit, with four C-note gates, as in Figure (3).

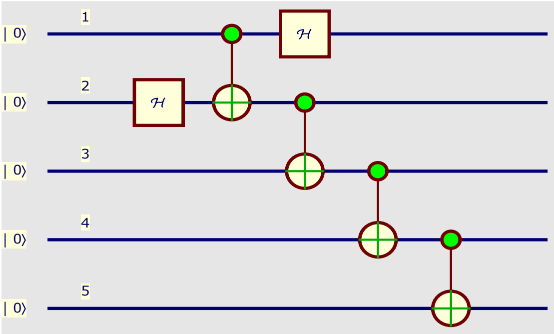

Figure (3): The circuit of entangled states for five qubits.

Since there are two HADAMARD gates, resulting in an output of four bases, as well as four c-not gates. The second qubit is subjected to HADAMARD in the first step: $H_2 \cdot |0>_2$

$$\frac{|0>_2}{\sqrt{2}} + \frac{|1>_2}{\sqrt{2}}$$

And since they will be in a single matrix and each qubit has a line that functions as an identity matrix, the first and second qubits must be combined.

$$\left(\frac{|0>_2}{\sqrt{2}} + \frac{|1>_2}{\sqrt{2}}\right) \otimes |0>_1$$

As may be seen, the following is the Dirac notation for (1,2) $\frac{|00\rangle}{\sqrt{2}} + \frac{|10\rangle}{\sqrt{2}}$. The identity matrix and the not gate matrix are combined to form the C-not matrix:

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

To obtain the identical the preparation was to multiply these two mattresses by two using the qubit that Alice had sent: $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$. This appears to be a bell measurement result, as can be seen: $\frac{|00>_{12}}{\sqrt{2}} + \frac{|11>_{12}}{\sqrt{2}}$. More efforts can be made to enhance the number of qubits we have and conduct future real-world experiments on this protocol to determine whether we might include more qubits in GHZ protocol in the practical application:

Tikrit Journal of Pure Science (2023) 28 (5): 65-73
Doi: https://doi.org/10.25130/tjps.v28i5.1389

$$\left(\frac{|00>_{12}}{\sqrt{2}}+\frac{|11>_{12}}{\sqrt{2}}\right)\otimes|0>_3$$

The aforementioned mathematical operations and Mathematica will be used to create a simulation on a classical computer of adding extra qubits to this circuit using another tenser product. Where it was obtained:

$$\frac{|000>_{123}}{\sqrt{2}}+\frac{|110>_{123}}{\sqrt{2}}$$

$$C^{\{2\}}[NOT_3].\left(\frac{|000>_{123}}{\sqrt{2}}+\frac{|110>_{123}}{\sqrt{2}}\right)$$

$$\left(\frac{|000>_{123}}{\sqrt{2}}+\frac{|111>_{123}}{\sqrt{2}}\right)\otimes|0\hat{}_4\rangle$$

$$C^{\{3\}}[NOT_4].\left(\frac{|0000>_{1234}}{\sqrt{2}}+\frac{|1110>_{1234}}{\sqrt{2}}\right)$$

$$\left(\frac{|0000>_{1234}}{\sqrt{2}}+\frac{|1111>_{1234}}{\sqrt{2}}\right)\otimes|0\hat{}_5\rangle$$

$$C^{\{4\}}[NOT_5].\left(\frac{|00000>_{12345}}{\sqrt{2}}+\frac{|11110>_{12345}}{\sqrt{2}}\right)$$

$$\frac{|00000>_{12345}}{\sqrt{2}}+\frac{|11111>_{12345}}{\sqrt{2}}$$

By setting all the qubits, this conclusion is finally reached (1,2,3,4,5). The following table will show each possible outcome if all inputs in state | 0> while changing the input one qubit at a time [Creating a New State by Applying Bell State to a Quantum System.

Table (1): computational result for the GHZ-Like Entanglement Circuit of five qubits

State	Input-output
0	$\|00000>$ $\quad \frac{1}{2}\|00000> + \frac{1}{2}\|01111> + \frac{1}{2}\|10000> + \frac{1}{2}\|11111>$
1	$\|00001>$ $\quad \frac{1}{2}\|00001> + \frac{1}{2}\|01110> + \frac{1}{2}\|10001> + \frac{1}{2}\|11110>$
2	$\|00010>$ $\quad \frac{1}{2}\|00011> + \frac{1}{2}\|01100> + \frac{1}{2}\|10011> + \frac{1}{2}\|11100>$
3	$\|00011>$ $\quad \frac{1}{2}\|00010> + \frac{1}{2}\|01101> + \frac{1}{2}\|10010> + \frac{1}{2}\|11101>$
4	$\|00100>$ $\quad \frac{1}{2}\|00111> + \frac{1}{2}\|01000> + \frac{1}{2}\|10111> + \frac{1}{2}\|11000>$
5	$\|00101>$ $\quad \frac{1}{2}\|00110> + \frac{1}{2}\|01001> + \frac{1}{2}\|10110> + \frac{1}{2}\|11001>$
6	$\|00110>$ $\quad \frac{1}{2}\|00100> + \frac{1}{2}\|01011> + \frac{1}{2}\|10100> + \frac{1}{2}\|11011>$
7	$\|00111>$ $\quad \frac{1}{2}\|00101> + \frac{1}{2}\|01010> + \frac{1}{2}\|10101> + \frac{1}{2}\|11010>$
8	$\|01000>$ $\quad \frac{1}{2}\|00000> - \frac{1}{2}\|01111> + \frac{1}{2}\|10000> - \frac{1}{2}\|11111>$

9	$\|01001>$	$\frac{1}{2}\|00001> - \frac{1}{2}\|01110> + \frac{1}{2}\|10001> - \frac{1}{2}\|11110>$
10	$\|01010>$	$\frac{1}{2}\|00011> - \frac{1}{2}\|01100> + \frac{1}{2}\|10011> - \frac{1}{2}\|11100>$
11	$\|01011>$	$\frac{1}{2}\|00010> - \frac{1}{2}\|01101> + \frac{1}{2}\|10010> - \frac{1}{2}\|11101>$
12	$\|01100>$	$\frac{1}{2}\|00111> - \frac{1}{2}\|01000> + \frac{1}{2}\|10111> - \frac{1}{2}\|11000>$
13	$\|01101>$	$\frac{1}{2}\|00110> - \frac{1}{2}\|01001> + \frac{1}{2}\|10110> - \frac{1}{2}\|11001>$
14	$\|01110>$	$\frac{1}{2}\|00100> - \frac{1}{2}\|01011> + \frac{1}{2}\|10100> - \frac{1}{2}\|11011>$
15	$\|01111>$	$\frac{1}{2}\|00101> - \frac{1}{2}\|01010> + \frac{1}{2}\|10101> - \frac{1}{2}\|11010>$
16	$\|10000>$	$\frac{1}{2}\|00000> + \frac{1}{2}\|01111> - \frac{1}{2}\|10000> - \frac{1}{2}\|11111>$
17	$\|10001>$	$\frac{1}{2}\|00001> + \frac{1}{2}\|01110> - \frac{1}{2}\|10001> - \frac{1}{2}\|11110>$
18	$\|10010>$	$\frac{1}{2}\|00011> + \frac{1}{2}\|01100> - \frac{1}{2}\|10011> - \frac{1}{2}\|11100>$
19	$\|10011>$	$\frac{1}{2}\|00010> + \frac{1}{2}\|01101> - \frac{1}{2}\|10010> - \frac{1}{2}\|11101>$
20	$\|10100>$	$\frac{1}{2}\|00111> + \frac{1}{2}\|01000> - \frac{1}{2}\|10111> - \frac{1}{2}\|11000>$
21	$\|10101>$	$\frac{1}{2}\|00110> + \frac{1}{2}\|01001> - \frac{1}{2}\|10110> - \frac{1}{2}\|11001>$
22	$\|10110>$	$\frac{1}{2}\|00100> + \frac{1}{2}\|01011> - \frac{1}{2}\|10100> - \frac{1}{2}\|11011>$
23	$\|10111>$	$\frac{1}{2}\|00101> + \frac{1}{2}\|01010> - \frac{1}{2}\|10101> - \frac{1}{2}\|11010>$
24	$\|11000>$	$-\frac{1}{2}\|00000> + \frac{1}{2}\|01111> + \frac{1}{2}\|10000> - \frac{1}{2}\|11111>$
25	$\|11001>$	$-\frac{1}{2}\|00001> + \frac{1}{2}\|01110> + \frac{1}{2}\|10001> - \frac{1}{2}\|11110>$

26	|11010>	$-\frac{1}{2}\|00011> + \frac{1}{2}\|01100> + \frac{1}{2}\|10011> - \frac{1}{2}\|11100>$
27	|11011>	$-\frac{1}{2}\|00010> + \frac{1}{2}\|01101> + \frac{1}{2}\|10010> - \frac{1}{2}\|11101>$
28	|11100>	$-\frac{1}{2}\|00111> + \frac{1}{2}\|01000> + \frac{1}{2}\|10111> - \frac{1}{2}\|11000>$
29	|11101>	$-\frac{1}{2}\|00110> + \frac{1}{2}\|01001> + \frac{1}{2}\|10110> - \frac{1}{2}\|11001>$
30	|11110>	$-\frac{1}{2}\|00100> + \frac{1}{2}\|01011> + \frac{1}{2}\|10100> - \frac{1}{2}\|11011>$
31	|11111>	$-\frac{1}{2}\|00101> + \frac{1}{2}\|01010> + \frac{1}{2}\|10101> - \frac{1}{2}\|11010>$

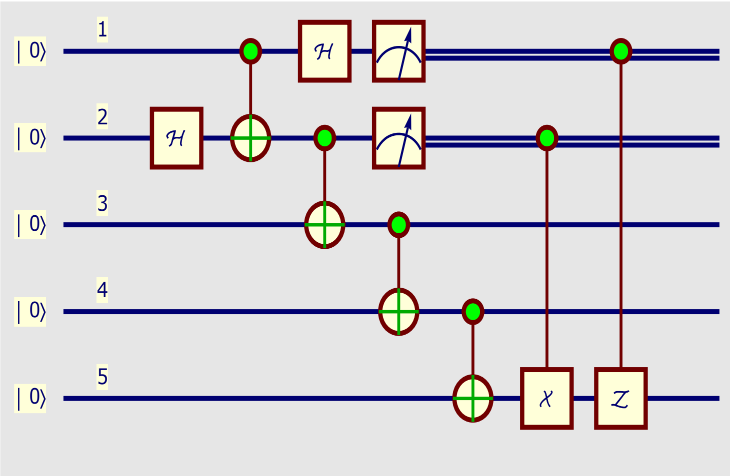

Figure (4): the entanglement state for four-basic states of five qubits after applying Bell state measurement.

Figure 4 shows the entanglement state having four-basis states composed of five qubits during quantum teleportation after applying Bell state measurement.

Conclusion

In this manuscript, the Mathematica program was used to find results and tables to transfer the 5-qubit state, which is a direct method for the teleportation of the entanglement states of the three bodies through a class similar to GHZ, which is a feature that users can only teleport the states of 2 qubits at a time in the best case of the previous protocols. In the second figure, because Alice and Bob could not retrieve the transmitted quantum states without Charlie's help, a third party was added as a controller. In the protocol of this manuscript, a programmable circuit consisting of 5 qubits is also proposed. When the input states of the first and second qubits are set to |0>, the amplitude of the output states has a positive value only because the Hadamard gate only operates on the first and second qubits. As for the case in which the input states of the first and second qubits are set both or only one of them two of the four-basis amplitudes of the output states will have a negative value.

The results obtained represent the teleportation state of an entangled state of five qubits and four bases as a quantum channel. BSMs have 32 possible outcomes, where the input changes in one qubit each time. The technique for creating entangled systems that will be useful in any application involving quantum computation can be applied in the future for bigger numbers of qubits. Instead of being employed for quantum teleportation, the proposed circuits' entangled states can be used for dense coding, quantum encrypting, and cryptography.

References

[1] Bennett, C.H.; Brassard, G.; Crepeau, C.; Jozsa, R.; Peres, A. and Wootters, W.K. (1993). Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. Physical Review Letters, 70:1895–1899.

[2] Dür, W.; Vidal, G. and Cirac, J.I. (2000). Three qubits can be entangled in two inequivalent ways. Physical Review A, 62:062314(1–12).

[3] Shi, B.S.; Jiang, Y.K. and Guo, G.C. (2000). Probabilistic teleportation of two-particle entangled state. Physics Letters A, 268(3):161–164.

[4] Wilde, M.M. (2019). Quantum information theory. 2nd ed., Cambridge University Press, arXiv:1106.1445 pp.774.

[5] Khatri, S., and Wilde, M.M. (2020). Principles of Quantum Communication Theory: A Modern Approach. arXiv. /abs/2011.04672: pp.971.

[6] Cai, X.F.; Yu, X.T.; Shi, L.H. and Zhang, Z.C. (2014). Partially entangled states bridge in quantum teleportation. Frontier of Physics, 9:646–651.

[7] Piattini, M.; Peterssen, G. and Pérez, R. (2021). Quantum computing: A new software engineering golden age. ACM SIGSOFT Software Engineering Notes, Association for Computing Machinery, 45(3):12–14.

[8] Piccitto, G.; Russomanno, A. and Rossini, D. (2022). Entanglement transitions in the quantum Ising chain: A comparison between different unravelings of the same Lindbladian. Physical Review B, 105(6):219901.

[9] Sisodia, M.; Shukla, A.; Thapliyal, K. and Pathak, A. (2017). Design and experimental realization of an optimal scheme for teleportation of an n-qubit quantum state. Quantum Information Processing, 16:292(1–11).

[10] Singha, R.; Sarkar, S.; Ghosh, A. and Roy, S. (2023). Signatures of topological surface state and unconventional magnetotransport properties in elemental Ruthenium. Advanced Quantum Technologies, 6(3):2200116(1–8).

[11] Sisodia, M. (2020). Design and Analysis of Communication Protocols using Quantum Resources. Ph.D. thesis, Declared Deemed to be University, Noida, India.

[12] Çelik, N. and Nüket, Ü. (2022). Producing maximally entangled Bell type states in an efficient way for quantum teleportation. Chinese Journal of Physics, 79:192–201.

[13] Tabatabaei, L.S. and Vakili, B. (2023). Bi-directional quantum teleportation of GHZ-like states. arXiv. /abs/2302.11300:1–13.