Tikrit Journal of Pure Science Vol. 28 (3) 2023

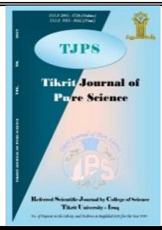
TJPS

https://doi.org/10.25130/tjps.v28i3.1430

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j



DOMINATING SET ON CHAIN OF FUZZY GRAPHS

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ARTICLE INFO.

Article history:

-Received: 15 / 9 / 2022
-Received in revised form: 18 / 9 / 2022
-Accepted: 13 / 10 / 2022
-Final Proofreading: 9 / 6 / 2023
-Available online: 25 / 6 / 2023

Keywords: chain fuzzy graphs, vertex identification, dominating set.

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ABSTRACT

In this paper, we define fuzzy graph chains, which comprise vertex identification. These fuzzy graphs are isomorphic fuzzy graphs, provide that after applying various features to the chain of fuzzy graphs, which as special fuzzy graph chain of C_5.

المجموعة المهيمنة على سلسلة البيانات المضببة

رسل حسن مجيد ، نبيل عز الدين عارف

قسم الرياضيات، كلية علوم الحاسوب و الرياضيات، جامعة تكريت، تكريت، العراق

الملخص

في هذا البحث، عرفنا سلسلة البيانات المضببة التي تتكون من تطابق البيانات بواسطة الرؤوس، و هذه البيانات تكون متشاكلة. و من ثم تم تطبيقها على بيانات الدارة ذات الخمس رؤوس. و مناقشة بعض الخواص عليها.

1. Introduction

One application tool in the field of mathematics is the fuzzy graph, which enables users to simply explain the link between any two conceptions. The concept of the graph with fuzziness and many graph theory analogs in the fuzziness ideas such as paths, cycles, and connectedness were first described by Rosenfeld in 1975[1]. In 2021, Mahmood and Ahmed introduced the vertex identification chain graphs " Schultz and Modified Schultz Polynomials for Vertex Identification Chain and Ring for Hexagon Graphs"[2]. Somasundaram introduced the concept of domination in graph with fuzziness in1998[3]. Nagoorgani in 2007 introduced domination depending on the strong edges in fuzzy graphs[4]. In this paper we introduced the idea of a chain of fuzzy graphs and a new type of dominating set which is an equal dominating set.

2. Basic concepts

A fuzzy graph denoted by $\mathfrak{G} = (\mathcal{V}, \varpi, \psi)$ on the crisp graph $G = (V, E)$ is a nonempty set $\mathcal{V}$ and two functions $\varpi: \mathcal{V} \to [0,1]$ and $\psi: \mathcal{V} \times \mathcal{V} \to [0,1]$ s.t. $\forall\, x, y \in \mathcal{V}$, the relation $\psi(x, y) \le \varpi(x) \wedge \varpi(y)$ is satisfied[5].

https://doi.org/10.25130/tjps.v28i3.1430

path P in graph with fuzziness $\mathfrak{G}$ is a collection of different vertices $\mathfrak{X}_0, \dots, \mathfrak{X}_n$, where $\mathfrak{X}_0 \neq \mathfrak{X}_n$, and $n \geq 2$ such that $\psi(\mathfrak{X}_{i-1}, \mathfrak{X}_i) > 0, i = 0, \dots, n$. We refer to the following pairs as the path's edges. Length of path is the number of edges[1]. A path P where $\mathfrak{X}_0 = \mathfrak{X}_n$ *and* $n \geq 3$ is a cycle. The weight of the weakest edge (the edge with least membership in path) is used to measure a path's strength. The strength of the connectedness between the vertices u and v, is the maximum strength of all paths linking them, and it is denoted by $CCON_{\mathfrak{G}}(u, v)$. A path connecting two vertices shows that they are linked[6]. A fuzzy graph $\mathfrak{G} = (\mathcal{V}, \varpi, \psi)$ is connected if $CCON_{\mathfrak{G}}(x, y) > 0, \forall\ x, y\ \in \mathcal{V}$. An edge (x, y) is strong in $\mathfrak{G}$ if $\mu(x, y) > 0$, and $\psi(x, y) \geq CCON_{\mathfrak{G}-(x,y)}(x, y)$[6]. An edge (u, v) in $\mathfrak{G}$ is called $\propto -srtong$ if $\psi(u, v) > CCON_{\mathfrak{G}-(u,v)}(u, v)$ [7]. An edge (u, v) in $\mathfrak{G}$ is called $\beta - srtong$ if $\psi(u, v) = CCON_{\mathfrak{G}-(u,v)}(u, v)$ [7]. If $(u, v) > 0$, then u and v are called neighbors. Also v is called a strong neighbor if edge (u, v) is strong [8].

Lemma 2.1[6] if $\psi(x, y) = \varpi(x) \wedge \varpi(y)$, then the edge (x, y) is strong.

3. Main Results

3.1. The Vertex- Identification Chain Fuzzy Graphs:

The following is a formal definition of a chain of fuzzy graphs.

Definition3.1.1: Assuming that $\{\mathfrak{G}_1, \mathfrak{G}_2, \dots, \mathfrak{G}_n\}$, be a set of pairwise disjoint fuzzy graphs with vertices $u_i, v_i \in \mathcal{V}(\mathfrak{G}_i)$ then the vertex-identification chain fuzzy graph , $C_{vf}(\mathfrak{G}_1, \mathfrak{G}_2, \dots, \mathfrak{G}_n \equiv C_{vf}(\mathfrak{G}_1, \mathfrak{G}_2, \dots, \mathfrak{G}_n: v_1 \cdot u_2; \dots; v_{n-1} \cdot u_n)$ of $\{\mathfrak{G}_i\}_{i=1}^{n-1}$ with respect to the vertices $\{v_i, u_{i+1}\}_{i=1}^{n-1}$ is the graph obtained from the fuzzy graphs $\mathfrak{G}_1, \mathfrak{G}_2, \dots, \mathfrak{G}_n$ by identifying the $v_i$vertex with the vertex u_{i+1} for all $i = 1,2 \dots n$. such that the vertex identification's weight value is $\varpi(w_i) = \max\{\varpi(v_i), \varpi(u_{i+1})\}$(denoted by $C_{vf}(C_n)$) (see Fig. 3.1)

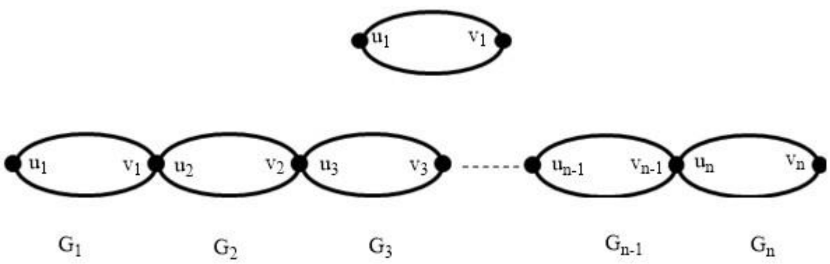


Fig. 3-1. Chain fuzzy graphs

3.2. Dominating set on Chain Fuzzy Graphs:

Definition 3.2.1[3]**:** A vertex a dominates other vertices in a fuzzy graph $\mathfrak{G} = (\mathcal{V}, \varpi, \psi)$. If $\psi(a, b) = \varpi(a) \wedge \varpi(b)$, for $a, b \in V$.

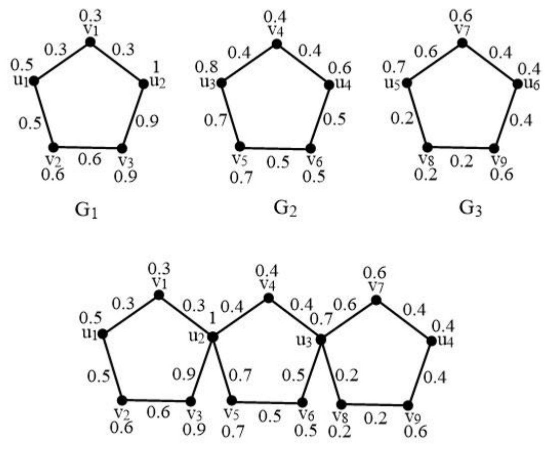
A set $\mathfrak{D} \subseteq \mathcal{V}$ is defined as a dominant set in $\mathfrak{G}$ if for each $b \in \mathcal{V} - \mathfrak{D}$, there exists $a \in \mathcal{V}$, so that, a dominates b. The dominance number of $\mathfrak{G}$ is a collection with the least possible cardinality. and is denoted by $\gamma(\mathfrak{G})$. A dominant set $\mathfrak{D}$ is called the minimal dominant set, if no proper subset of S is a dominating set .

Definition 3.2.2 [6] : A fuzzy graph vertex is considered to be an isolated vertex if $\psi(a, b) < \varpi(a) \wedge \varpi(b), a, b \in \mathcal{V} - \mathfrak{D}$.

Definition3.2.3[9]**:** A subset $\mathfrak{D}$ of $\mathcal{V}$ is called 2-dominant set of $\mathfrak{G}$, if for any vertex $u \in \mathcal{V} - \mathfrak{D}$ two or more strong neighbors may be found in $\mathfrak{D}$.The 2-dominance number of a fuzzy graph $\mathfrak{G}$ signified by $\gamma_2(\mathfrak{G})$ is the smallest 2-dominating set of $\mathfrak{G}$'s cardinality.

Remark 3.2.1: Every fuzzy graph in a chain of fuzzy graphs is a cycle, which means there is no edge between any dominating vertex.

Example 3.2.1: In this example, all the edges are strong, which means there is no isolated vertex since isolated vertex does not dominate another vertex.



$C_{vf}(C_5)$

Fig 3-2 $C_{vf}(C_5)$

In this example, there are many dominating sets. One of these dominating sets is $\mathfrak{D} = \{u_1, u_2, u_3, u_4\}$ the vertices in $\mathfrak{D}$ that dominate the other vertices. $\mathfrak{D}$ is the set of vertex identification.

The domination number is $\left(C_{vf}(C_n)\right) = 2.6$.

https://doi.org/10.25130/tjps.v28i3.1430

Remark 3.2.2: In a chain of fuzzy graphs, every minimal dominating set (domination number) must contain the vertex identifications.

Theorem 3.2.1: In $C_{vf}(C_n)$, if $n \le 6$. Then $\left(C_{vf}(C_n)\right) = \sum_{i=1} \varpi(u_i)$.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs with vertex identification $v_i \in \mathfrak{D}$. Let $n \le 6$. Since $n \le 6$, then, every vertex in $\mathcal{V} - \mathfrak{D}$ adjacent to the vertex identifications, which means the vertex identifications $v_i \in \mathfrak{D}$ dominate the vertices in $-\mathfrak{D}$. From the definition of domination number, we get the vertex identifications as the minimal dominating set of $C_{vf}(C_n)$. Then $\gamma\left(C_{vf}(C_n)\right) = \sum_{i=1} \varpi(v_i)$.

In last example $\gamma\left(C_{vf}(C_n)\right) = 2.6$, This is the summation of all vertex identifications' fuzzy membership.

Remark 3.2.3: In $C_{vf}(C_n)$, if $n > 6$. Then the domination number is the minimal dominating set contain the vertex identifications.

Theorem 3.2.2 [9]**:** Each 2-dominant set of a fuzzy graph $\mathfrak{G}$ is the dominant set of $\mathfrak{G}$.

Theorem 3.2.3: In $C_{vf}(C_n)$, if $n \le 4$. Then every 2-domination number is domination number.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs with vertex identification $v_i \in \mathfrak{D}$. Let $n \le 4$. Let $\mathfrak{D}$ be a 2- dominating set, then, $\forall u_i \in \mathcal{V} - \mathfrak{D}$, has at least two neighbors in $\mathfrak{D}$. Since $n \le 4$, every vertex is adjacent to two of the vertex identifications, which means the vertex identifications are the minimal 2-dominant set. Since every Each 2-dominant set is a dominant set, every 2-domination number is a domination number.

Definition 3.2.4 [4]**:** Consider $\mathfrak{G} = (\mathcal{V}, \varpi, \psi)$ to be a fuzzy graph. if $\psi(a, b) = \varpi(a) \wedge \varpi(b)$ and $d_{\mathfrak{G}}(a) \ge d_{\mathfrak{G}}(b)$**,** a strongly dominates b in $\mathfrak{G}$**,** For any two vertices $a, b \in \mathcal{V}$**.** Similarly, in $\mathfrak{G}$**,** a weakly dominates b if $\psi(a, b) = \varpi(a) \wedge \varpi(b)$ and $d_{\mathfrak{G}}(a) \le d_{\mathfrak{G}}(b)$. If any vertex in a subset $\mathcal{V} - \mathfrak{D}$ has at minimum one vertex in the subset $\mathfrak{D}$ that is strongly dominant, the subset is considered to be a strong dominant set of . Similarly, if any vertex in a subset $\mathcal{V} - \mathfrak{D}$ at least one vertex in the collection $\mathfrak{D}$ weakly dominates, then the subset $\mathfrak{D}$ is a weakly dominating set of $\mathfrak{G}$. The strong dominance number (denoted by $\gamma_s(\mathfrak{G})$) is a strong dominant set's minimal fuzzy number of vertices. Similarly, the weak dominance number (denoted by $\gamma_w(\mathfrak{G})$) is the minimal fuzzy number of vertices of a weak dominant set.

Example 3.2.2: In Fig3-2 .

$\mathfrak{D} = \{u_1, u_2, u_3, u_4\}$, all the vertices in $\mathcal{V} - \mathfrak{D}$ which is strongly dominant by a vertex in $\mathfrak{D}$. $\mathfrak{D}$ is a strong dominant set and the strong domination number is $\gamma_s\left(C_{vf}(C_5)\right) = 2.6$.

$\mathfrak{D}_1 = \{v_1, v_3, v_4, v_6, v_7, v_9\}$, all the vertices in $\mathcal{V} - \mathfrak{D}$ is weakly dominant by a vertex in $\mathfrak{D}$. $\mathfrak{D}$ is a weak dominant set and the weak domination number is $\gamma_w\left(C_{vf}(C_5)\right) = 3.3$.

Theorem 3.2.4: In a chain of fuzzy graphs if $n \le 6$. Then the set of the vertex identifications is a strong domination number.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs with vertex identifications $v_i \in \mathfrak{D}$. Let $n \le 6$. Let $\mathfrak{D}$ be the strongly dominant set. The degree of the vertex identification is greater than other vertices. Hence $n \le 6$ then the vertex identification $v_i \in \mathfrak{D}$, is dominating other vertices strongly in $\mathcal{V} - \mathfrak{D}$. The vertex identification is the minimal cardinality of a strong dominant set.

Then the set of the vertex identifications is a strong domination number.

Theorem 3.2.5: In a chain of fuzzy graphs if $4 \le n \le 6$. Then

$\gamma_s(C_{vf}(C_n)) \le \gamma_w\left(C_{vf}(C_n)\right)$.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs with vertex identification u_i. Let $4 \le n \le 6$. The vertex identification is a strong dominance number, and it is the minimal number of vertices of a strong dominating set.

In every fuzzy graph in the chain, there exist at least two weakly dominating vertex, which means the cardinality of a weak dominating set is greater than the cardinality of a strong dominating set.

Then $\gamma_s\left(C_{vf}(C_n)\right) \le \gamma_w\left(C_{vf}(C_n)\right)$.

Theorem 3.2.6: In a chain of fuzzy graphs,

$\gamma_s\left(C_{vf}(C_n)\right) + \gamma_w\left(C_{vf}(C_n)\right) \le O\left(C_{vf}(C_n)\right)$.

Proof : Clearly, the sum of the strong domination number of a chain of fuzzy graphs and the weak domination number is less than or equal to the order of the chain.

Example3.2.3: From the example

$\gamma_s\left(C_{vf}(C_5)\right) = 2.6$, $\gamma_w\left(C_{vf}(C_5)\right) = 3.3$, $O\left(C_{vf}(C_5)\right) = 9.2$

$\gamma_s\left(C_{vf}(C_n)\right) + \gamma_w\left(C_{vf}(C_n)\right) = 2.6 + 3.3 = 5.9 < 9.2$.

Definition 3.2.5: A set $\mathfrak{D} \subseteq V$, is an equal dominating set if every vertex in $\mathfrak{D}$ has the same number of neighbors in $\mathfrak{D} - \mathcal{V}$. The equal dominance number is the minimum number of vertices of an equal dominant set of $\mathfrak{G}$ (denoted by $\gamma_e(\mathfrak{G})$).

Example 3.2.4: In Fig.3-2, $\mathfrak{D} = \{u_1, u_2, u_3, u_4\}$, is an equal dominating set, also, $\mathfrak{D}_1 = \{v_1, v_3, v_4, v_6, v_7, v_9\}$, is an equal dominating set. $\mathfrak{D}$ is the minimum an equal dominating set, $\gamma_e\left(C_{vf}(C_5)\right) = 2.6$.

Remark3.2.4: In a chain of fuzzy graphs, every fuzzy graph is a cycle, so every vertex has the same number of neighbors.

Theorem 3.2.7: In a chain of fuzzy graphs, every equal- dominant set is a dominant set.

https://doi.org/10.25130/tjps.v28i3.1430

Proof : let $\mathfrak{D}$, be an equal- dominant set, every vertex in $\mathfrak{D}$ has the same number of neighbors in $\mathcal{V}-\mathfrak{D}$. Any vertex $\mathcal{v} \in \mathcal{V}-\mathfrak{D}$ there are a vertex $\mathcal{u} \in \mathfrak{D}$, s.t., $\mathcal{u}$ dominates $\mathcal{v}$. Then $\mathfrak{D}$ is a dominant set.

Theorem 3.2.8: If $n = 5$, in a chain of fuzzy graphs, then strong dominating set is an equal dominating set.

Proof : let $\mathfrak{D}$, be a strong dominant set. Every vertex in $\mathcal{V}-\mathfrak{D}$, is strongly dominant be a vertex in $\mathfrak{D}$. For every vertex $\mathcal{v} \in \mathcal{V}-\mathfrak{D}$, there is a vertex $\mathcal{u} \in \mathfrak{D}$, such that, $\mathcal{u}$ strongly dominates $\mathcal{v}$. Since every vertex in the cycle has the same number of neighbors. then $\mathfrak{D}$ is an equal dominating set.

Theorem3.2.9: In $C_{vf}(C_5)$, the vertex identification is an equal domination number.

Proof : Let $C_{vf}(C_5)$, be a chain of fuzzy graph with vertex identification $\mathcal{u}_i$. Let $\mathfrak{D}$, minimum an equal dominating set. $n = 5$, the vertex identification $\mathcal{u}_i$ dominates every vertex in $\mathcal{V}-\mathfrak{D}$, and every vertex identification has the same number of neighbors. Then the vertex identification is an equal domination number.

Remark 3.2.5: In $C_{vf}(C_5)$, an equal domination number is

$$\gamma_e\left(C_{vf}(C_5)\right) = \sum_{i=1}^{n} \varpi(\mathcal{u}_i).$$

4. Conclusions

In this paper, we discussed the relationship between a dominating set, a strong dominating set, and a weak dominating set on a chain of fuzzy graphs. We presented a new kind of dominating set called an equal dominating set. The vertex identification satisfies the conditions of dominating, strong dominating, and an equal dominating set.

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