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M Nano –Separation Axioms $M_N - T_i$ Spaces

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ABSTRACT

The aim of the research is to define new types of M –nano –separation axioms then named $M_N - T_i$ ($M_N - semi - T_i$, $M_N - pre - T_i$), where $i = 0,1,2$, Also there are some theorems and examples shown the relationship among these types of spaces.

بديهيات الفصل النانوية من النمط M

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قسم الرياضيات، كلية التربية للعلوم الصرفة، جامعة تكريت، تكريت ، العراق.

الملخص

الهدف من البحث هو تعريف أنواع جديدة من بديهيات الفصل من النمط M_N تم تسميتها $M_N - T_i$ ($M_N - semi - T_i$, $M_N - pre - T_i$),where $i = 0,1,2$ مع برهنة بعض النظريات بحيث أظهرت الأمثلة العلاقة بين هذه الأنواع من الفضاءات.

1. Introduction and preliminaries

The notion of M-open sets in topological spaces were introduced by El-Ma introduced ghrabi and Al-Juhani [5] in 2011 and studied some of their properties.

In 2013Thivagar M.lelis[4] introduce idea of Nano-topological space with respect to a subset $X$of universe U which is defined as an upper and lower approximation of X. The element of N-topological space is called a Nano-open sets. In 2019 A. Padma, M. Saraswathi and A. Vadivel, [1] introduced Nano M-Separation axioms. In 2022 Mohmmed N.H and Shihab Ali A. [3] introduced definition of Nano-M-open set. The purpose of this paper is to discuss the separation axioms via Nano-M-open sets such as Nano-M-T_i spaces where $= 0,1,2$, Nano-M-regular and Nano-M- Normal spaces are introduced and some of the properties are discuss spaced .

2. Some Types $M_N - T_i$-spaces, where $i = 0,1,2$

Definition 2.1[2]: A $Nts(U, T_R(x))$ is called

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1- $M_N - T_0$ space, if for any two points $L \neq m, \exists M - No$set containing only of them.

2- $M_N - T_1$ space, if for any two points $L \neq m, \exists M - No$ sets such that one containing$L$ but not $m$and the other containing m but not L.

3- $M_N - T_2$ space, if for any two points $L \neq m$, $\exists$ disjoint $M - No$sets G and H for every $L \neq m \in U$ such that $L \in G$ and $f \in H$.

Definition 2.2[1]: A function $f: (U, T_R(x)) \to (Q, T_R(y))$ is said to be

1- M_N −continuous if for each $N - c$ set a of Q, the set $f^{-1}(a)$ is $M_N - c \subseteq U$.

2- M_N −irresolute if for each a $M_N - c$ set of Q, then $f^{-1}(a)$ is $M_N - c$ in U.

3. M_N −Separation Axioms

In this section, we define and study the ideal some types of $M_N - T_i$space and obtained some of their basic results.

Definition 3.1: A space $(U, T_{R(x)})$ is said to be:

1- M-Nano semi-T_0 space ($M_N - S - T_0$ space) if $\exists x, y \in U, \exists G$ is M_{NS} −open set such that either $x \in G, y \notin G$ or $x \notin G, y \in G$.

2- M-Nano pre T_0-space ($M_N - pre - T_0$ space) , $\exists x, y \in U, \exists$ an M_N pre-open set containing only one of them.

Theorem 3.4: If $U_{R(x)} \neq U, U_{R(x)} \neq L_{R(x)}, L_{R(x)} \neq \emptyset$, then $(U, T_{R(x)}) M_N - S - T_0(M_N - pre - T_0)$ for any $a, b \in U$ such that $a \in U_{R(x)}$ and $b \in [U_{R(x)}]^c$ or $a \in B_{R(x)}$ and $b \in L_{R(x)}$.

Proof: To prove That $(U, T_{R(x)})$ is $M_N - S - T_0$.

In this case $T_{R(x)} = \{U, \phi, L_{R(x),} U_{R(x)}, B_{R(x)}\}$. Let$a \in U_{R(x)}$ and $b \in \{U_{R(x)}\}^c$ then $U_{R(x)}$ is Nano-openset containing a and $b \notin U_{R(x)}$. That is U is $M_N - S - T_0$ for any a and b. Suppose $a \in L_{R(x)}$ and $b \in B_{R(x)}$. Since $L_{R(x)}$ is N-o set containing a and $b \notin L_{R(x)}$. That is U is $M_N - S - T_0$ for any a and b.

Corollary 3.5: If $U_{R(x)} = U$ and $L_{R(x)} \neq \phi$, then U is$M_N - S - T_0(M_N - pre - T_0), \forall a, b \in U$, such that $a \in U_{R(x)}$ and $b \in B_{R(x)}$.

Corollary3.6: Then $U_R(x) = L_{R(x)} = x$, if U is $M_N - S - T_0(M_N - pre - T_0), \forall a, b \in U$, such that $a \in U_{R(x)}$ and $b \in [U_{R(x)}]^c$

Theorem 3.7: Let$(U, T_{R(x)})$ be NTs, then $\forall$ M_N−T_0-Space is $M_N - S - T_0(resp\ M_N - p - T_0)$.

Proof: Let U be M_N−T_0-Space, x and y be twodistinct points of U, as $U$is $M_N T_0$. $\exists$ M_{No} set such that $x \in G$ and $y \notin G$, Since every M_{No} set is M_N-S(resp $M_N - pre$)-openand hence $G$is$M_N - S(resp\ M_N - pre)$o-s such that $x \in G$ and $\notin G$. U is $M_N - S - T_0(resp\ M_N - pre - T_0)$ space. But the converse of the theorem need not be true in general.

Example 3.8: Let $U = \{1,2,3,4\}, \frac{U}{R} = \{\{1\}, \{2,4\}, \{3\}\}, X = \{1,2\}$ and $T_{R(x)} = \{U, \emptyset, \{1\}, \{1,2,4\}, \{2,4\}\}$ be a NT on U. Wehave

$M_N so(U, X) = \{U, \phi, \{1\}, \{1,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\},$

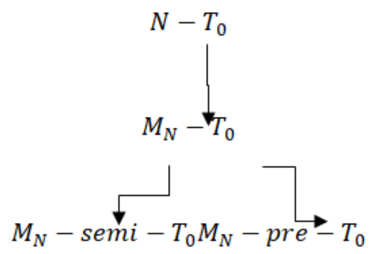
$M_N po(U, X) = \{U, \phi, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}\{1,3,4\}\}$

.

1-Let $x = \{1,3\}$ and $y = \{3\}$ then it is $M_N - S - T_0$ space but not $M_N - T_0$ space.

2-Let $x = \{2\}$ and $y = \{3\}$ then it is is $M_N - per - T_0$ space but not $M_N - T_0$ −space.

Diagram (1)



Definition 3.9.:A space $(U, T_{R(x)})$ is said tobe:

1- M_N-Semi−T_1-space for every $x \neq y \in U, \exists$ two open set such that one containing x but not$y$ and the other containing y but not x.

2- M_N-pre −T_1-space $\forall x, y \in U, \exists$ two M_N-pre-open set such that one containing x but not y and the other containing y butnot x.

Theorem 3.10: Every $M_{N-}T_1$ is $M_N - S - T_1$.

Theorem 3.11: Every $M_N - T_1$ is M_N −pre−T_1.

Proof: Let U be $M_{N-}T_1$ space and $x \neq y$ in U. Then $\exists$ distinct $M - No$ sets G and H such that $x \in G$ and $y \in H$. As every M-N-O set is $M_N - preT_1(resp\ M_N - semi\ T_1)$ open G and H are distinct $M_N - pre(res\ M_N - S)$ open sets such that $x \in G$ and $y \in H$. Converse of above theorem is need not be true in general.

Example 3.12: Let $U = \{1,2,3,4\}, U/R_1 = \{\{1\}, \{2,4\}, \{3\}\}, x_1 = \{1,2\}\ and\ T_{R(x_1)} = \{U, \phi, \{1\}, \{1,2,4\}, \{2,4\}\}$ be a NT on U, we have

$MNSo(U, x_1) = \{U, \emptyset, \{1\}, \{1,3\}, \{2,4\}, \{1,2,4\}, \{2,3,4\}\}$

$MNpo(U, x_1) = \{U, \emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}\{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}\}$

$U/R_2 = \{\{1,2\}, \{3\}, \{4\}\}, x_2 = \{2,3\}$ and $T_{R(x_2)} = \{U, \phi, \{3\}, \{1,3,4\}\}, \{1,4\}\}$.

We have

$MNSo(U, x_2) = \{U, \phi, \{3\}, \{1,4\}, \{2,3\}, \{1,2,4\}, \{1,3,4\}\}$

$MNpo(U, x_2) = \{U, \phi, \{1\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,3,4\}\}$.

1- Let $x = \{1,3\}$ and $y = \{1,2\}$ then it is $M_N - S - T_1$ space but not $M_N - T_1$ space.

2- Let $x = \{2\}$ and $y = \{3\}$ then it is $M_N - pre - T_1$ space but not $M_N - T_1$ space.

Theorem 3.13: If $U_{R(x)} = U$ and $L_{R(x)} \neq \phi$, and $U_R \neq L_{R(x)}$then U is $M_N - S - T_1(M_N - pre - T_1)$ for any $a, b \in U$s.t.$a \in L_{R(x)}$ and $b \in B_{R(x)}$.

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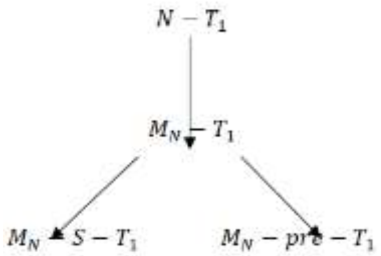
Proof: To prove $M_N - pre - T_1$. Let $a, b \in U$ such that $a \in L_{R(x)}$ and $b \in B_{R(x)}$, since $L_{R(x)}$ and $B_{R(x)}$ are $M_N - o$ Sets and $L_{R(x)} \cap B_{R(x)} = \emptyset$ This implies to that is U is $M_N - pre - T_1(M_N - S - T_1)$ for any a and b.

Theorem 3.14: Let $(U, T_{R(x)})$ be a NTs then $\forall M_N - pre - T_1(res\, M_N - S - T_1)$ space is $M_N - pre - T_0$ space.

Proof: Let U be $M_N - pre - T_1(res\, M_N - S - T_1)$ space and a and b be two distinct points of U, as U is $M_N - pre - T_1(res. M_N - S - T_1)$ space $\exists\, M_N - pre(resp. M_N - S)$ open set Gsuch that$a \in G$ and $b \in G$, since every M_N −open set is $M_N - pre(res. M_N - S)$ open sets such that $a \in G$ and $b \notin G \Longrightarrow U$ is $M_N - pre - T_0$.

Diagram (2)



Definition 3.15: Aspace $(U, T_{R(x)})$ is said to be:

1- $M_N - semi - T_2$ space($M_N - S - T_2$ space)that there exist two disjoint open sets G and V for every $x \neq y \in U$ such that $x \in G$ and $y \in V$.

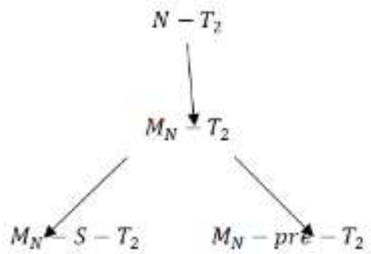
2- $M_N - pre - T_2$ space ($M_N - p - T_2$ space) $\exists\, x, y \in U \exists$two disjoint M-N open sets G and $V$for every $x \neq y \in U$ s.t. $x \in G$ and $y \in V$.

Theorem 3.16.: Every $M_N - T_2$ space is $M_N - p - T_2(resp. M_N - S - T_2)$ space.

Proof: Let U be $M_N - T_2$ space and let $x \neq y$ in U, then $\exists$ disjoint $M_N - O$sets G and H such that $x \in G, y \in H$. As every $M_N - Oset$ is $M_N - pre(resp. M_N - S)$ open G and H are disjoint $M_N - pre(resp. M_N - S)$ open sets such that $x \in G, y \in H$. Converse of above theorem is need not be true in general.

Example 3.17: From Example 3.12. Let $x = \{1\}, y = \{3\}, x, y \in U, x \neq y$, then it is clear that $x \in G, g \notin G$ and $y \in H, x \notin H$, then we can say that it is $M_N - pre - T_0$ space but not $M_N - p - T_2(resp. M_N - S - T_2)$ space.

Diagram (3)



Theorem 3.18: Let $(U, T_{R(x)})$ be a $N - T$ space then for each $M_N - p - T_2(resp. M_N - S - T_2)$ space is $M_N - pre - T_0$ space.

Proof: Let U be $M_N - p - T_2(resp. M_N - S - T_2)$space and x, y be two distinct points of U, as U is $M_N - p - T_2(resp. M_N - S - T_2)$ space $\exists M_N - pre(res. M_N - S)$ open set G such that $x \in G, y \notin G$, since every $M_N - open$ set is $M_N - p\ (resp. M_N - S)$ open and hence G is $M_N - pre(resp. M_N - Semi)$ open set such that $x \in G, y \notin G \Rightarrow U$ is$M_N - p - T_0$.

Diagram (4)

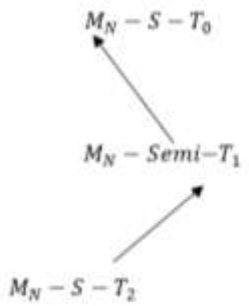
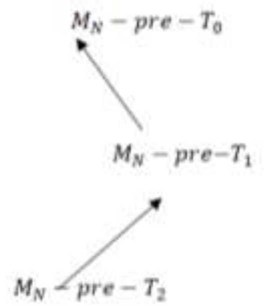


Diagram (5)



Theorem 3.19: If $f: (U, T_{R(x)}) \to (Q, T_{R(y)})$ is injectiveM_N sets function and Q is a $N - T_0$ space then U is a $M_N - S - T_0$ space.

Proof: Suppose that a and b is any points in $U, a \neq b$ and f injective such that $f(a), f(b)$ in $Q, f(a) \neq f(b)$, by using Remark (2) (every N-open set is $M_N - semi$ open($M_N - pre$) set. Since Q is$M_N - S - T_0$ space, $\exists$ a NOS G in Q containing $f(a)$ butnot $f(b)$, Againsince f is M_N sets, $f^{-1}(G)$ is a$M_N - OS$in U containing$a$ but not b. Therefore$U$ is a $M_N - S - T_O$ space.

Theorem 3.20: If $f: (U, T_{R(x)}) \to (Q, T_{R(y)})$ is an injective $M_N$irr form and Q is a $M - NST_0$ then U is a $M_N - S{-}T_0$.

Proof: Suppose Q is$M_N - S - T_0$ space. Let a and b be any two distinct points in U. Since f is injective $f(a)$ and $f(b)$ are distinct points in$Q$. Since$Q$ is a$M_N - S{-}T_0$ space, $\exists$ a M_N −open set G in Q containing $f(a)$ butnot $f(b)$. Again, since$f$ is $M_N - Sirr, f^{-1}(G)$is a$M - NOS$in$U$ containing $a$but not b therefore$U$ is a $M_N - S - T_0$ space.

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Theorem 3.21: If $f:(U,T_{R(x)}) \rightarrow (Q,T_{R(y)})$ is an injective $M_N\, irr$ function and Q is a $M_N - S - T_1(M_N - pre - T_1)$ space. Then U is also a $M_N - S - T_1(M_N - pre - T_1)\ space$.

Proof: Let a and b be pair of distinct points in U. since f is injective, $\exists$ two distinct points c and d of Qsuch that $f(a) = c, f(b) = d$. Since Q is a $M_N - S - T_1$ space, $\exists\ M - No$ sets G and W in Qsuchthat $c \in G, d \notin G$ and $c \notin W, d \in W$. That is $a \in f^{-1}(G), a \notin f^{-1}(W)$ and $b \in f^{-1}(W), b \notin f^{-1}(G)$, since f is $M_N - irr$ function $f^{-1}(G)$ and $f^{-1}(W)$ are $M - No$ sets in U. Thus, for any two distinct points a, b of $U \exists\ M - No$sets $f^{-1}(G)$ and $f^{-1}(W)$such that $a \in f^{-1}(G), a \notin f^{-1}(W)$ and $b \in f^{-1}(W)$ and $b \notin f^{-1}(G)$. Therefore U is $M_N - S - T_1$space.

Theorem 3.22: If $f:(U,T_{R(x)}) \rightarrow (Q,T_{R(y)})$ is a M-N sets injective and Q is a $N - T_1$ −space. Then U is a $M_N - pre - T_1(M_N - S - T_1)$ space.

Proof: for any two distinct points a, b of $U \exists$ two distinct points c and d of Qsuch that $f(a) = c, f(b) = d$, since Q is a $N - T_1$ −space,$\exists$ a $M_N oSG$ and Win Q such that $a \in G, b \notin G$ and $a \notin W, b \in W$. That is $a \in f^{-1}(G), a \notin f^{-1}(W)$ and $b \in f^{-1}(W), b \notin f^{-1}(G)$. Since$f$ is a M-Ncts function $f^{-1}(G)$ and $f^{-1}(W)$ are M_N −open sets in U. Thus, for any two distinct points a, b of $U \exists M_N oSf^{-1}(G)$ and $f^{-1}(W)$ Such that $a \in f^{-1}(G), a \notin f^{-1}(W)$ and $b \in f^{-1}(W), b \in f^{-1}(G)$. Therefore, U is a $M_N - pre - T_1$ space.

Theorem 3.23: If $f:(U,T_{R(x)}) \rightarrow (Q,T_{R(y)})$ is a $M_N cts$ injection and Q is a $N - T_2$ −space, then U is a $M_N - S - T_2(M_N - p - T_2)$ space.

Proof: for any two distinct points $a$and b of $U, \exists$ two distinct points c, d of Q suchthat $f(a) = c, f(b) = d$, since Q is $N - T_2$ space, $\exists$ a disjoint Nos G and Win$Q$such that $a \in G$ and $b \in W$. That is $a \in f^{-1}(G)$ and $b \in f^{-1}(W)$. Since$f$is $a\ N - Mcts$ function, $f^{-1}(G)$ and $f^{-1}(W)$ are $M_N oS$ in U. Further as f is injective $f^{-1}(G) \cap f^{-1}(W) = f^{-1}(G \cap W) = f^{-1}(\phi) = \phi$. Thus, for any two distinct points$a$ and b of $U, \exists M_N oSf^{-1}(G)$ and $f^{-1}(W)$such that $a \in f^{-1}(G)$ and $b \in f^{-1}(W)$. Therefore U is $M_N - S - T_2$ space.

Theorem 3.24: If $f:(U,T_{R(x)}) \rightarrow (Q,T_{R(y)})$ is an injective, M-N irr function and Q is a $M_N - p - T_2(M_N - S - T_2)$ space, then Uisalso a $M_N - p - T_2(resp. M_N - S - T_2)$ space.

Proof: Let a and b be a pair of distinct pointsin$U$. Since$f$ is injective are distinct in$Q$such that $f(a)$ and $f(b)$ are distinct points in Q. $\exists$two distinct points$c$ and $d$of Q such that $f(a) = c, f(b) = d$. Since$Q$ is a $M_N - T_2$ space, $\exists$ a disjoint $M - NoSG$ and W in Q such that $a \in G, b \in W$. That is $a \in f^{-1}(G), b \in f^{-1}(W)$. Since$f$ is a $M_N - irr$ function $f^{-1}(G)$ and $f^{-1}(W)$ are disjoint $M_N os$ in$U$. Thus, for any two distinct points a and b of $U, \exists$ disjoint $M_N osf^{-1}(G)$ and $f^{-1}(w)$ such that $a \in f^{-1}(G)$ and $b \in f^{-1}(W)$, Therefore U is$M_N - p - T_2$ space.

Definition 3.25: A $M_N - T_S(U,T_{R(x)})$ be called M-Nano-regular space (briefly $M_N - R - S$). If for any $M_N - cl$ −set W and a point p such that $p \notin W \exists$ a disjoint $M_N - o$ sets M and Qsuch that $WCM$and $p \in Q$

Theorem 3.26: If $U_{R(x)} = U, L_{R(x)} \neq \phi, U_{R(x)} \neq L_{R(x)}$,then the space U is$M_N - R$ −space.

Proof: since $U_{R(x)} = U$ and $L_{R(x)} \neq \emptyset$, then $T_{R(x)} = \{U, \phi, L_{R(x)}, B_{R(x)}\}$ the $M_N - cl$ sets are $U, \emptyset, [L_{R(x)}]^C, [B_{R(x)}]^C$. $\forall M_N - O$ set in $T_{R(x)}$ are $M_N - cl$ set in U, that is $L_{R(x)} = [B_{R(x)}]^C$,$B_{R(x)} = [L_{R(x)}]^C$.

Thus,U is extremely disconnected space.Therefore,U is $M_N - R$ −space.

Lemma 3.27: when $U_{R(x)} \neq U, L_{R(x)} = U_{R(x)} = x$.Then the space $U$is not $M_N - R$ −space.

Theorem 3.28: If $U_{R(x)} \neq U, L_{R(x)} \neq \emptyset$, then the space is not $M_N - R$ −space.

Proof: when $U_{R(x)} \neq U, L_{R(x)} \neq \phi$, then $T_{R(x)} = \{U, \phi, L_{R(x)}, U_{R(x)}, B_{R(x)}\}$. The M_N-cl-sets in U, are $U, \phi, [L_{R(x)}]^C, [U_{R(x)}]^C$ and $[B_{R(x)}]^C$. All these $M_N - cl$-sets contained only in$U$, so that if taking any point p out of any $M_N - cl$ −set, then the only$M_N - o$ −set which contain the$M_N - cl$ −set is U, and $p \in U$. Hence $U$is not $M_N - R$ −space.

Definition 3.29:A $M_N - R$ −space which also $M_N - T_1$space be called $M_N - T_3$ space.

Example 3.30: Let $U = \{1,2,3,4\}$ with $U/R = \{\{1,2\}, \{3,4\}\}$ and $X = \{1,2,3\}$. Then $T_{R(x)} = \{U, \emptyset, \{1,2\}, \{3,4\}\}$, the $M_N - cl$ −sets are $U, \phi, \{1,2\}$ and $\{3,4\}$. Then U is $M_N - T_1$ for $\{1,2\}$ and $\{3,4\}$. And U is $M_N - R$ Space. U is $M_N - T_3$ space for $\{1,2\}$ and $\{3,4\}$.

Definition 3.31:A space U be called M_N-Normal iff $\forall$ two disjoint $M_N - cl\ W_1, W_2 \subseteq U$,$\exists$ disjoint M_N-o-set G and Hsuch that $W_1 \subset G, W_2 \subset H$.

Theorem 3.32: If $U_{R(x)} = U, U_{R(x)} \neq L_{R(x)}$ and $L_{R(x)} \neq \emptyset$ then the space U is$M_N - N$ −space.

Proof: when $U_{R(x)} = U$ and $L_{R(x)} \neq \emptyset$. then $T_{R(x)} = \{U, \emptyset, B_{R(x)}, L_{R(x)}\}$, the $M_N - cl$-sets are $U, \emptyset, \{B_{R(x)}\}^C$ and $\{U_{R(x)}\}^C$.

Suppose $W_1 = [B_{R(x)}]^C$ and $W_2 = [L_{R(x)}]^C, W_1 \cap W_2 = \emptyset$. Since $W_1 cL_{R(x)}$ and $W_2 cB_{R(x)}$. Since $L_{R(x)}$ and $B_{R(x)}$are disjoint $N_N - o$-sets. Hence is M_N-Normal-space.

Lemma 3.33: If $U_{R(x)} \neq U, L_{R(x)} \neq \emptyset$, then the space is $M_N - N$ −space.

Proof: In this case there are only three cl-sets except U. these cl-sets are $[L_{R(x)}]^C, [B_{R(x)}]^C$ and $[U_{R(x)}]^C$, since all these cl-sets are not disjoint. So that U is $M_N - N$ −space.

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Definition 3.34: Aspace $\left(U, T_{R(x)}\right)$ be called $M_N - S - N$ −space($M_N - pre - Normal$) if for any two disjoint $M_N - S - cl(res. M_N - pre - cl)$ sets W_1 and W_2 ∃ adisjoint $M_N - S - O(res. M_N - pre - o)$ sets G and H such that $W_1 \subset G$ and $W_2 \in H$, and symbolized be $M_N - S - Normal(res. M_N - pre - N)$.

Theorem 3.35: ∀ M_N −extremely disconnected space the following are true:

a) Every $M_N - N$ −space is $M_N - S - N$.

b) Every $M_N - N$ −space is $M_N - p - N$.

Proof: a) If U be extremely disconnected space and U is $M_N - N$ −space [Theorem 3.32]. Since $T_{R(x)} = M_N - S - o$.

b) Since every M_N-open is M_N-pre then U is M_N-pre.

Definition 3.36: A $M_N - N$ −space which also $M_N - T_1$ space be called $M_N - T_4$ space $(M_N - T_4)$.

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