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THE DEGREE ON CHAIN OF FUZZY GRAPHS

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ABSTRACT

In this paper, we discussed some types of degrees on chains of fuzzy graphs. also introduced a new type of degree, which is called an average degree.

درجة الرؤوس على سلسلة البيانات المضببة

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قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة تكريت، تكريت، العراق

الملخص

في هذا البحث، ناقشنا بعض انواع درجات الرؤوس على سلسلة البيانات المضببة، وتم تقديم نوع جديد من درجات الرؤوس والتي تسمى متوسط درجة الرأس.

Introduction

The fuzzy graph is a mathematical application tool that helps users quickly describe the connection between any two notions. Rosenfeld introduced the notion of the fuzzy graph and several fuzzy counterparts of concepts from graph theory, like paths, cycles, and connectedness, in 1975[1]. Nagoor Gani and Basheer Ahmed compared the relationships between degree, order, and size of fuzzy graphs while examining the characteristics of different kinds of degree, order, and size of fuzzy graphs [2]. Mahmood and Ahmed presented Schultz and Modified Schultz Polynomials for Vertex

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Identification Chain and Ring for Hexagon Graphs in 2021[3]. This paper aims to introduce a new type of degree. It also compares the relationship between different types of degrees.

2. Basic Concepts:

The fuzzy graph $\mathbb{G} = (V, \sigma, \mu)$ on the crisp graph $G = (V, E)$ is formed of the nonempty set V and two functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\forall\, x, y \in V$, the relation $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ is fulfilled [4].

The fuzzy set μ is referred to the fuzzy edge set and σ the fuzzy vertex set. In a fuzzy graph $\mathbb{G}$, a path P is a collection of vertices with distinct values of $x_0, \dots, x_n$ where $x_0 \neq x_n$ and $n \geq 2$ such that $\mu(x_{i-1}, x_i) > 0, i = 0, \dots, n$.The edges of the path are the pairs that follow [4]. The number of edges equals the length of the path. a P -path where $n \geq 3$and $x_0 = x_n$are cycles. A path's strength is defined by the weight of its weakest edge. The strength of connectivity between two vertices, indicated by $CCON_{\mathbb{G}}(x, y) or\ \mu^{\infty}(x, y)$is the shortest path among all those linking vertices x and y A path connecting two vertices denotes that they are linked. If $CCON_{\mathbb{G}}(x, y) > 0, \forall\, x, y \in V$ a fuzzy graph $\mathbb{G} = (V, \sigma, \mu)$is connected [5].

3. Main results:

3.1. The Vertex- Identification Chain Fuzzy Graphs:

A chain of fuzzy graphs is defined formally as follows.

Definition3.1.1: Assuming that $\{\mathbb{G}_1, \mathbb{G}_2, \dots, \mathbb{G}_n\}$, is a collection of pairwise disjoint fuzzy graphs with vertices $u_i, v_i \in V(\mathbb{G}_i)$ the fuzzy graph created, $C_{vf}(\mathbb{G}_1, \mathbb{G}_2, \dots, \mathbb{G}_n \equiv C_{vf}\ (\mathbb{G}_1, \mathbb{G}_2, \dots, \mathbb{G}_n: v_1 \cdot u_2; \dots; v_{n-1} \cdot u_{n)}$ from $\{\mathbb{G}_i\}_{i=1}^{n-1}$ is known as the vertex identification a chain of fuzzy graphs and such that the vertex identification's membership value is $\sigma(w_i) = \max\{\sigma(v_i), \sigma(u_{i+1})\}$ (denoted by $C_{vf}(c_n)$) (see Fig. 3.1), and in which:

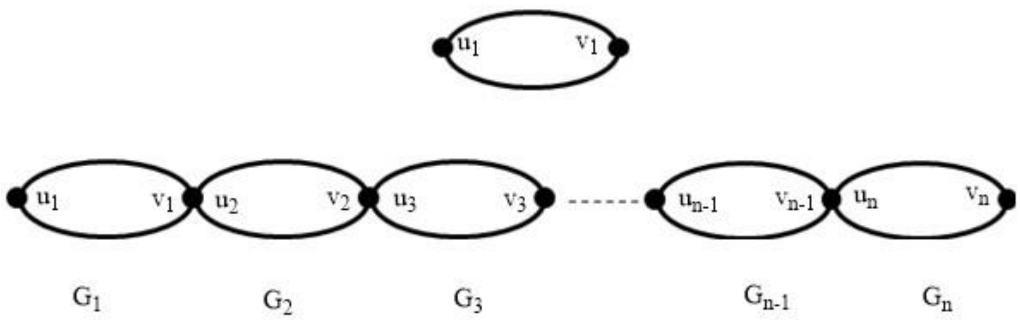

Fig. 3-1. Chain fuzzy graphs

3.2. Order and size of $C_{vf}(c_n)$:

Definition3.2.1:[6] Let $\mathbb{G} = (V, \sigma, \mu)$ be a fuzzy graph. The order $O(\mathbb{G})$ and the size $S(\mathbb{G})$ of $\mathbb{G}$ are defined by:

$$O(\mathbb{G}) = \sum_{a \in V} \sigma(a) \quad , \quad S(\mathbb{G}) = \sum_{(a,b) \in E} \mu(a, b)$$

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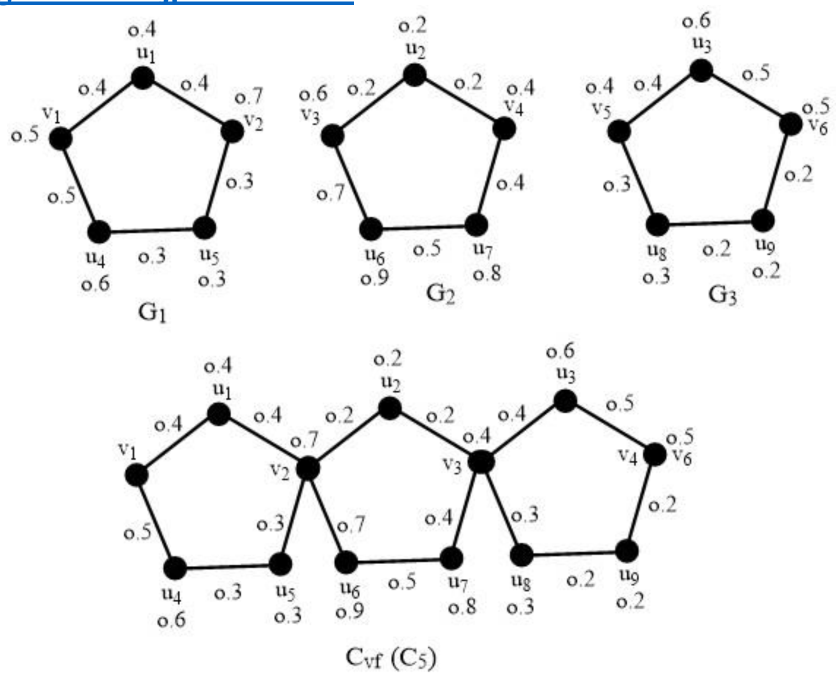

Fig.2-2 $\mathbf{C_{vf}(c_5)}$

Example3.2.1: From Fig.2-2 the order of $C_{vf}(c_5)$ is $O(C_{vf}(c_5)) = 6.4$, and the size of $C_{vf}(c_5)$ is $S(C_{vf}(c_5)) = 5.5$.

Theorem3.2.1 : In chain fuzzy graphs,

$S(C_{vf}(\mathbb{G}_i)) \leq O(C_{vf}(\mathbb{G}_i))$.

Proof : Suppose that $C_{vf}(\mathbb{G}_i)$ be a chain of fuzzy graphs. Let $u, v \in V(\mathbb{G}_i)$, and $uv \in E(\mathbb{G}_i)$. From the definition of fuzzy graph

$\mu(u, v) \leq \sigma(u) \wedge \sigma(v),\ u, v \in V$ that means the membership values of the vertices are greater than the membership values of the edges.

Since $S\left(C_{vf}(\mathbb{G}_i)\right) = \sum_{(a,b)\in E} \mu(a, b)$, and

$O\left(C_{vf}(\mathbb{G}_i)\right) = \sum_{a\in V} \sigma(a)$.

Then $S(C_{vf}(\mathbb{G}_i)) \leq O(C_{vf}(\mathbb{G}_i))$.

3.3. Degree and Average Degree of $\mathbf{C_{vf}(c_n)}$:

We will now introduce a new type of degree and discuss some known types of degrees and the relationships between them.

Definition3.3.1:[4] Let $\mathbb{G} = (V, \sigma, \mu)$ be a fuzzy graph. The degree of a vertex a is $d_{\mathbb{G}} = \sum_{(a,b)\in E} \mu(a, b)$ for $(a, b) \in E$.

Definition3.3.2:[4] The minimum degree of $\mathbb{G}$ is $\delta(\mathbb{G})$ is defined by $\delta(\mathbb{G}) = \min\{\, d_{\mathbb{G}}: a \in V\}$.

Definition3.3.3:[4] The maximum degree of $\mathbb{G}$ is $\Delta(\mathbb{G})$ is defined by $\Delta(\mathbb{G}) = \max\{\, d_{\mathbb{G}}: a \in V\}$.

Example3.3.1 : In Fig(2-2), the degree of the vertices are:

$$d_{C_{vf}(C_5)}(v_1) = 0.9, d_{C_{vf}(C_5)}(v_2) = 1.5,$$
$$d_{C_{vf}(C_5)}(v_3) = 1.3,\ d_{C_{vf}(C_5)}(v_4) = 0.7,$$
$$d_{C_{vf}(C_5)}(u_1) = 0.7, d_{C_{vf}(C_5)}(u_2) = 0.9, d_{C_{vf}(C_5)}(u_3) = 0.9,$$
$$d_{C_{vf}(C_5)}(u_4) = 0.8, d_{C_{vf}(C_5)}(u_5) = 0.6, d_{C_{vf}(C_5)}(u_6) = 1.2,$$
$$d_{C_{vf}(C_5)}(u_7) = 0.9, d_{C_{vf}(C_5)}(u_8) = 0.5, d_{C_{vf}(C_5)}(u_9) = 0.4\,.$$

the maximum degree of $C_{vf}(C_5)$is $\Delta\left(C_{vf}(C_5)\right) = 1.5$, and the minimum degree of $C_{vf}(C_5)$ is $\delta\left(C_{vf}(C_5)\right) = 0.4$.

Remark3.3.1: The degree of the vertex identifications is greater than any other vertices.

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Theorem3.3.1: In the chain of fuzzy graphs, $\Delta(\mathbb{G})$ is the degree of one of the vertex identifications.

Proof : Let $C_{vf}(\mathbb{G}_i)$ be a chain of fuzzy graphs, and let w_i be the vertex identifications and v_i be the other vertices in $C_{vf}(\mathbb{G}_i)$.

Since every vertex identification links two graphs, there are four edges incident on the vertex identification. The degree of vertex identification is greater than the degree of other vertices.

By the definition of $\Delta(\mathbb{G})$, then, $\Delta(\mathbb{G})$ is the degree of one of the vertex identifications.

Definition3.3.4:[5] The neighborhood degree of a vertex u is denoted by:

$deg_N(u) = \sum_{v\in N(u)} \sigma(v)$. The minimum $\delta_N(\mathbb{G})$ and the maximum $\Delta_N(\mathbb{G})$ neighborhood degrees of fuzzy graph are given by :

$\delta_N(\mathbb{G}) = \min\{deg_N(u): u \in V\}$ and $\Delta_N(\mathbb{G}) = \max\{deg_N(u): u \in V\}$.

Example3.3.2: In Fig.(2-2) The neighborhood degrees of the vertices are :

$deg_N(v_1) = 1, deg_N(v_2) = 1.8, deg_N(v_3) = 1.9,$
$deg_N(v_4) = 1.3, deg_N(u_1) = 1.2,$
$deg_N(u_2) = 1.2, deg_N(u_3) = 0.9,$
$deg_N(u_4) = 1.1, deg_N(u_5) = 1.3$
$, deg_N(u_6) = 1.5, deg_N(u_7) = 1.3,$
$deg_N(u_8) = 0.6, deg_N(u_9) = 0.8.$

From the example $\Delta_N\left(C_{vf}(C_5)\right) = 1.9, \delta_N\left(C_{vf}(C_5)\right) = 0.6$.

Remark3.3.2: $\delta_N\left(C_{vf}(C_5)\right) \le \Delta_N\left(C_{vf}(C_5)\right)$.

Remark3.3.3: $\Delta_N\left(C_{vf}(C_5)\right)$ is the neighborhood degree of one of the vertex identifications.

Corollary3.3.1: In the chain of fuzzy graphs,

$$\delta\left(C_{vf}(C_5)\right) \le \delta_N\left(C_{vf}(C_5)\right) \le \Delta\left(C_{vf}(C_5)\right) \le \Delta_N\left(C_{vf}(C_5)\right) \le S(C_{vf}(c_5)) \le O(C_{vf}(c_5)) .$$

Proof : Let $C_{vf}(C_5)$ be a chain of fuzzy graphs.

Since $\delta\left(C_{vf}(C_5)\right) = \min\{ d_{\mathbb{G}}: v \in V\}$.

Since $\delta_N\left(C_{vf}(C_5)\right) = \min\{ deg_N: v \in V\}$, and $deg_N(u) = \sum_{v\in N(u)} \sigma(v)$. From the condition of fuzzy graph

$\mu(u,v) \le \sigma(u) \wedge \sigma(v)$.

We get $\delta\left(C_{vf}(C_5)\right) \le \delta_N\left(C_{vf}(C_5)\right) \ldots.(1)$

Since $\delta_N\left(C_{vf}(C_5)\right) = \min\{ deg_N: v \in V\}$, and

$\Delta\left(C_{vf}(C_5)\right) = \max\{ d_{\mathbb{G}}: v \in V\}$.

$\min\{ deg_N: v \in V\} \le \max\{ d_{\mathbb{G}}: v \in V\}$.

Then $\delta_N\left(C_{vf}(C_5)\right) \le \Delta\left(C_{vf}(C_5)\right)$….(2)

From the definition of $\Delta\left(C_{vf}(C_5)\right) = \max\{ d_{\mathbb{G}}: v \in V\}$, and

$\Delta_N\left(C_{vf}(C_5)\right) = \max\{deg_N(u): u \in V\}$.

$\Delta\left(C_{vf}(C_5)\right)$ depends on the sum of membership values of all edges incident to u. $\Delta_N\left(C_{vf}(C_5)\right)$ depends on the sum of the membership values of all neighbors.

Then $\Delta\left(C_{vf}(C_5)\right) \le \Delta_N\left(C_{vf}(C_5)\right)$….(3) .

Now $\Delta_N\left(C_{vf}(C_5)\right) = \max\{deg_N(u): u \in V\}$.

$S(C_{vf}(c_5))= \sum_{(u,v)\in E} \mu(u,v)$ [the sum of membership values for all edges in the chain].

Then $\Delta_N\left(C_{vf}(C_5)\right) \le S(C_{vf}(c_5))$….(4).

Now, from theorem definition of size and order

We get $S(C_{vf}(c_5)) \le O(C_{vf}(c_5))$….(5)

From (1), (2), (3), (4), (5) we get :

$$\delta\left(C_{vf}(C_5)\right) \le \delta_N\left(C_{vf}(C_5)\right) \le \Delta\left(C_{vf}(C_5)\right) \le \Delta_N\left(C_{vf}(C_5)\right) \le S(C_{vf}(c_5)) \le O(C_{vf}(c_5)) .$$

Definition3.3.5: Let $\mathbb{G} = (V, \sigma, \mu)$ be a fuzzy graph, the average degree of a vertex u is

$deg_v(u) = \frac{d_{\mathbb{G}}(u)}{the\ number\ of\ N(u)}$.

Definition3.3.6: The minimum average degree of $\mathbb{G}$ is $\delta_v(\mathbb{G})$ is defined by $\delta_v(\mathbb{G}) = \min\{ deg_v: a \in V\}$.

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Definition3.3.7: The maximum average degree of $\mathbb{G}$ is $\Delta_v(\mathbb{G})$ is defined by $\Delta_v(\mathbb{G}) = \max\{\, deg_v : a \in V\}$.

Example3.3.3:

$deg_v(v_1) = 0.45, deg_v(v_2) = 0.375,$
$deg_v(v_3) = 0.325, deg_v(v_4) = 0.35,$
$deg_v(u_1) = 0.35, deg_v(u_2) = 0.45, deg_v(u_3) = 045,$
$deg_v(u_4) = 0.4, deg_v(u_5) = 0.3, deg_v(u_6) = 0.6,$
$deg_v(u_7) = 0.45, deg_v(u_8) = 0.25, deg_v(u_9) = 0.2.$

In this example $\Delta_v\left(C_{vf}(C_5)\right) = 0.6$, which is the degree of u_6.

$\delta_v\left(C_{vf}(C_5)\right) = 0.325$, which is the degree of the vertex identification v_3.

Remark3.3.4: The average degree of the vertex identifications is smaller than any other vertices.

Theorem3.3.2: In a chain of fuzzy graphs, $\delta_v\left(C_{vf}(C_5)\right)$, is the average degree of one of the vertex identifications.

Proof: Let $C_{vf}(\mathbb{G}_i)$ be a chain of fuzzy graphs, and let w_i be the vertex identifications and v_i be the other vertices in $C_{vf}(\mathbb{G}_i)$.

Since every vertex identification links two graphs, there are four edges incident on the vertex identification. Since

$$deg_v(u) = \frac{d_{\mathbb{G}}(u)}{the\ number\ of\ N(u)}.$$

The average degree of vertex identification is smaller than the average degree of other vertices. By the definition of $\delta_v\ (C_{vf}(C_5))$, then, $\delta_v(C_{vf}(C_5))$ is the average degree of one of the vertex identifications.

Theorem3.3.3: In a chain of fuzzy graphs,

1. $\delta_v\left(C_{vf}(C_n)\right) < \delta\left(C_{vf}(C_n)\right)$.
2. $\Delta_v\left(C_{vf}(C_5)\right) < \Delta\left(C_{vf}(C_5)\right)$.

Proof:

1. It is clear, since $\delta_v\left(C_{vf}(C_n)\right) = \min\{\, deg_v : a \in V\}$, and $deg_v(u) = \frac{d_{\mathbb{G}}(u)}{the\ number\ of\ N(u)}$.

$\delta\left(C_{vf}(C_n)\right) = \min\{\, d_{\mathbb{G}} : a \in V\}$, and $d_{\mathbb{G}}(u) = \sum_{(u,v)\in E} \mu(u,v)$.

Then $\delta_v\left(C_{vf}(C_n)\right) < \delta\left(C_{vf}(C_n)\right)$.

2. Clearly, since $\Delta_v\left(C_{vf}(C_n)\right) = \max\{\, deg_v : a \in V\}$, and

$deg_v(u) = \frac{d_{\mathbb{G}}(u)}{the\ number\ of\ N(u)}$.

$\Delta\left(C_{vf}(C_n)\right) = \max\{\, d_{\mathbb{G}} : a \in V\}$, and $d_{\mathbb{G}}(u) = \sum_{(u,v)\in E} \mu(u,v)$.

Then $\Delta_v\left(C_{vf}(C_5)\right) < \Delta\left(C_{vf}(C_5)\right)$.

4. Conclusions:

In this paper, we discussed the relationship between some type of degrees, and we introduced a new type of degree called average degree. The degree of vertex identification is greater than other vertices. In average degree the vertex identification is the minimum average degree.

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