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On Soft S_p-Closed and Soft S_p-Open Sets with Some Applications

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ABSTRACT

In this article, the concept of a soft S_p-closed set is introduced. Its relationships with some other types of soft sets are explored and discussed. In addition, via soft S_p-closed sets and soft S_p-open sets, the concepts soft S_p-neighborhood, soft S_p-limit point, soft S_p-derived, soft S_p-interior, soft S_p-closure, and soft S_p-boundary are introduced and investigated.

حول المجموعات الناعمة المغلقة والمفتوحة من النمط S_p مع بعض تطبيقاتها

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الملخص

في هذه المقالة، يتم تقديم مفهوم **المجموعة المغلقة** الناعمة من النمط S_p. يتم ايجاد ومناقشة علاقاتها مع بعض الأنواع الأخرى من المجموعات الناعمة. بالإضافة إلى ذلك، من خلال المجموعات **المغلقة** الناعمة من النمط S_p والمجموعات المفتوحة الناعمة من النمط S_p ، المفاهيم (الجوار، نقطة الغاية، المشتقة، الداخلية، انغلاق، و الحدودية) الناعمة من النمط S_p تم تقديمها والتحقيق فيها.

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1- Introduction and Preliminaries

The concepts and information proposed in [1] are used in this article. Molodtsov defined soft sets [2] as follows: Assume X is a universe set, $\mathfrak{P}(X)$ is the power of X, and $\mathcal{P}$ is a set of parameters. A pair $(\mathcal{E},\mathcal{P})=\{(e,\mathcal{E}(e)): e\in\mathcal{P},\mathcal{E}(e)\in\mathfrak{P}(X)\}$ is known as a soft set over X, where $\mathcal{E}:\mathcal{P}\rightarrow\mathfrak{P}(X)$ is a function. The family of all soft sets over the universal set X with the set of parameters $\mathcal{P}$ is indicated by $\tilde{S}S(X,\mathcal{P})=\tilde{S}S(\tilde{X})$. In particular, $(X,\mathcal{P})$ is indicated by $\tilde{X}$. Maji et al. [3], was defined a null soft set, indicated by $\widetilde{\emptyset}$, if $\mathcal{E}(e)=\emptyset,\forall\, e\in\mathcal{P}$ and an absolute soft set, indicated by $\tilde{X}$, if $\mathcal{E}(e)=X,\forall\, e\in\mathcal{P}$ and the soft complement of a soft set $(\mathcal{E},\mathcal{P})$ is indicated by $\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})=(\mathcal{E}^c,\mathcal{P})$ where $\mathcal{E}^c:\mathcal{P}\rightarrow\mathfrak{P}(X)$ is a function defined as $\mathcal{E}^c(e)=X-\mathcal{E}(e),\forall\, e\in\mathcal{P}$. The soft union of $(\mathcal{E}_\vartheta,\mathcal{P})\tilde{\in}\tilde{S}S(\tilde{X})$, $\forall\,\vartheta\in\aleph$ is a soft set $(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S(\tilde{X})$, where $\mathcal{E}(e)=\tilde{\cup}_{\vartheta\in\aleph}\mathcal{E}_\vartheta(e),\forall\, e\in\mathcal{P}$, $\aleph$ is a random collection of index and the soft intersection of $(\mathcal{E}_\vartheta,\mathcal{P})\tilde{\in}\tilde{S}S(\tilde{X}),\forall\,\vartheta\in\aleph$ is a soft set $(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S(\tilde{X})$, where $\mathcal{E}(e)=\tilde{\cap}_{\vartheta\in\aleph}\mathcal{E}_\vartheta(e),\forall\, e\in\mathcal{P}$, were defined in. A soft point [4] $(\mathcal{E},\mathcal{P})$ is a soft set defined as $\mathcal{E}(e)=\{x\}$ and $\mathcal{E}(\acute{e})=\emptyset,\forall\,\acute{e}\in\mathcal{P}\setminus\{e\}$, we indicated by $\tilde{e_x}$ such that $\tilde{e_x}=(e,\{x\})$, where $x\in X$ and $e\in\mathcal{P}$. $\tilde{e_x}\tilde{\in}(B,\mathcal{P})$, if $e\in\mathcal{P}$ and $\{x\}\subseteq B(e)$. The family of all soft points over X is indicated by $\tilde{S}P(\tilde{X})$. The idea of soft topological space ($\tilde{S}TS$) over X was defined in [5] is $(\tilde{X},\tilde{\tau},\mathcal{P})$ (simply, $\tilde{X}$), where $\tilde{\tau}\tilde{\subseteq}\tilde{S}S(\tilde{X})$ is known as "soft topology" on $\tilde{X}$, if $\widetilde{\emptyset},\tilde{X}\tilde{\in}\tilde{\tau}$, and $\tilde{\tau}$ is closed under finite soft intersection and arbitrary soft union. The members of $\tilde{\tau}$ are referred to as soft open sets. The soft complements of every soft open or members of $\tilde{\tau}^c$ are known as soft closed sets [6]. A soft set $(\mathcal{E},\mathcal{P})$ that is both soft open and soft closed is referred to as a soft clopen set. The family of all soft clopen sets in $\tilde{X}$ is indicated by $\tilde{S}CO(\tilde{X})$. Let $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. Then, the soft closure of $(\mathcal{E},\mathcal{P})$ is $\tilde{s}cl(\mathcal{E},\mathcal{P})=\tilde{\cap}\{(C,\mathcal{P}):(C,\mathcal{P})\tilde{\in}\tilde{\tau}^c and\ (\mathcal{E},\mathcal{P})\tilde{\subseteq}(C,\mathcal{P})\}$. Clearly, $\tilde{s}cl(\mathcal{E},\mathcal{P})$ is the smallest soft closed set contains $(\mathcal{E},\mathcal{P})$ [5] and the soft interior of $(\mathcal{E},\mathcal{P})$ is $\tilde{s}int(\mathcal{E},\mathcal{P})=\tilde{\cup}\{(M,\mathcal{P}):(M,\mathcal{P})\tilde{\in}\tilde{\tau}\ and\ (M,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})\}$. Clearly, $\tilde{s}int(\mathcal{E},\mathcal{P})$ is the largest soft open set contained in $(\mathcal{E},\mathcal{P})$ [6]. The triple $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ is a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ where $Z\subseteq X$, $\tilde{\tau}_Z=\{(\mathcal{E}_Z,\mathcal{P})=\tilde{Z}\tilde{\cap}(\mathcal{E},\mathcal{P});(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{\tau}\}$ is known as "the soft relative topology" on $\tilde{Z}$, and $\mathcal{E}_Z(e)=\tilde{Z}\tilde{\cap}\mathcal{E}(e)$, for all $e\tilde{\in}\mathcal{P}$ [5].

In this paper, we define soft S_p-closed sets as the soft complements of soft S_p-open sets. Thus, soft S_p-closed sets can be defined via soft semi-closed sets and soft pre-open sets. We show that the class of soft S_p-closed sets strictly placed between the classes of soft S_c-closed sets and soft semi-closed sets. We introduce the basic properties of soft S_p-closed sets and their relationships with some other types of soft sets. Also, we establish the connections between a soft topological space and its soft subspace topologies through the utilization of soft S_p-closed sets. In addition to these, we introduce and investigate the notions of soft S_p-neighborhood, soft S_p-limit point, soft S_p-derived, soft S_p-interior, soft S_p-closure, and soft S_p-boundary of soft sets. Finally, we provide some basic relationships between a soft topological space and its soft subspaces in terms of soft S_p-interior and soft S_p-closure notions.

Further important terms and results are pointed out in the coming sections.

Definition 1.1. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$ is known as a soft semi-open [7] (resp., soft pre-open [8], soft α-open [9], soft b-open [10], soft β-open [11] and soft regular open [8]) set, if $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))$ (resp., $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$, $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}int(\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P})))$, $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))\tilde{\cup}\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$, $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}cl(\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P})))$, and $(\mathcal{E},\mathcal{P})=\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$).

The family of all soft semi (resp., pre, α, b, β, and regular) open sets in $\tilde{X}$ is indicated by $\tilde{S}SO(\tilde{X})$ (resp., $\tilde{S}PO(\tilde{X})$, $\tilde{S}\alpha O(\tilde{X})$, $\tilde{S}bO(\tilde{X})$, $\tilde{S}\beta O(\tilde{X})$ and $\tilde{S}RO(\tilde{X})$).

Definition 1.2. The soft complement of a soft semi (resp., pre, α, b, β, and regular) open set is known as soft semi-closed [7] (resp., pre [8], α [9], b [10], β [11], and regular [12]) closed. The family of all soft semi (resp., pre, α, b, β, and regular) closed sets in $\tilde{X}$ is indicated by $\tilde{S}SC(\tilde{X})$ (resp., $\tilde{S}PC(\tilde{X})$, $\tilde{S}\alpha C(\tilde{X})$, $\tilde{S}bC(\tilde{X})$, $\tilde{S}\beta C(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$).

Definition 1.3. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$ is known as soft S_p [1] (resp., $\tilde{S}S_c$ [13]) -open set, if $(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}SO(\tilde{X})$ and $\forall\,\tilde{e_x}\tilde{\in}(\mathcal{E},\mathcal{P})$, there is $(W,\mathcal{P})\tilde{\in}\tilde{S}PC(\tilde{X})$ (resp., $\tilde{\tau}^c$) such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$. The family of all soft S_p (resp., $\tilde{S}S_c$) -open subsets of $\tilde{X}$ is indicated by $\tilde{S}S_pO(\tilde{X})$ (resp., $\tilde{S}S_cO(\tilde{X})$).

The soft complement of a $\tilde{S}S_c$-open set is known as $\tilde{S}S_c$-closed [13] and the family of all $\tilde{S}S_c$-closed sets in $\tilde{X}$ is indicated by $\tilde{S}S_cC(\tilde{X})$.

Definition 1.4. ([14], [8], [9], [10], [15], [13]) Let $\mathfrak{Z}\in$ {semi, pre, α, b, β, S_c}, $\mu\in\{S,P,\alpha,b,\beta,S_c\}$ and $(\mathcal{E}_1,\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. Then,

(1) The soft $\mathfrak{Z}$-closure of $(\mathcal{E}_1,\mathcal{P})=\tilde{\cap}\{(C,\mathcal{P}):(C,\mathcal{P})\tilde{\in}\tilde{S}\mu C(\tilde{X}),(\mathcal{E}_1,\mathcal{P})\tilde{\subseteq}(C,\mathcal{P})\}$. They are indicated by $\tilde{s}scl(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}pcl(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}\alpha cl(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}bcl(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}\beta cl(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}S_ccl(\mathcal{E}_1,\mathcal{P})$.

(2) The soft $\mathfrak{Z}$-interior of $(\mathcal{E}_1,\mathcal{P})=\tilde{\cup}\{(M,\mathcal{P}):(M,\mathcal{P})\tilde{\in}\tilde{S}\mu O(\tilde{X}),(M,\mathcal{P})\tilde{\subseteq}(\mathcal{E}_1,\mathcal{P})\}$. They are indicated by $\tilde{s}sint(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}pint(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}\alpha int(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}bint(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}\beta int(\mathcal{E}_1,\mathcal{P})$, $\tilde{s}S_cint(\mathcal{E}_1,\mathcal{P})$.

Definition 1.5. [16] Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. A soft point $\tilde{e_x}\tilde{\in}\tilde{S}P(\tilde{X})$ is known as a

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(1) Soft semi-neighborhood of $\tilde{e_x}$, if there is $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$. The soft semi-neighborhood system at $\tilde{e_x}$, indicated by $\tilde{\mathbb{N}}_s(\tilde{e_x})$, is the family of all its soft semi-neighborhood.

(2) Soft semi-limit point of $(\mathcal{E},\mathcal{P})$, if $(W,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$, $\forall\, (W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ containing $\tilde{e_x}$. The family of all soft semi-limit points of $(\mathcal{E},\mathcal{P})$ is named soft semi-derived set of $(\mathcal{E},\mathcal{P})$ and is indicated by $\tilde{s}sD(\mathcal{E},\mathcal{P})$.

(3) Soft semi-boundary point of $(\mathcal{E},\mathcal{P})$, if $\forall\, (W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ containing $\tilde{e_x}$, we have $(W,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) \neq \tilde{\emptyset}$ and $(W,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})) \neq \tilde{\emptyset}$. The family of all soft semi-boundary points of $(\mathcal{E},\mathcal{P})$ is indicated by $\tilde{s}sBd(\mathcal{E},\mathcal{P})$.

Definition 1.6. [17] Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. The soft θ-interior of $(\mathcal{E},\mathcal{P})$ is the soft set $\tilde{s}\theta int(\mathcal{E},\mathcal{P}) = \tilde{\cup}\{(M,\mathcal{P}); \tilde{s}cl(M,\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$ and $(M,\mathcal{P}) \tilde{\in} \tilde{\tau}\}$. The soft set $(\mathcal{E},\mathcal{P})$ is known as a soft θ-open if $\tilde{s}\theta int(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P})$. The soft complement of a soft θ-open set is known as soft θ-closed. The family of all soft θ-closed sets in $\tilde{X}$ is indicated by $\tilde{S}\theta C(\tilde{X})$.

Definition 1.7. [18] A soft point $\tilde{e_x}$ is known as a soft semi-θ-adherent point of $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, if $\tilde{s}scl(W,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) \neq \tilde{\emptyset}$, for any $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. The set of all soft semi-θ-adherent points of $(\mathcal{E},\mathcal{P})$ is called soft semi-θ-closure of $(\mathcal{E},\mathcal{P})$ indicated by $\tilde{s}s\theta cl(\mathcal{E},\mathcal{P})$. The soft set $(\mathcal{E},\mathcal{P})$ is called soft semi-θ-closed, if $(\mathcal{E},\mathcal{P}) = \tilde{s}s\theta cl(\mathcal{E},\mathcal{P})$.

The soft complement of a soft semi-θ-closed set is called soft semi-θ-open. The family of all soft semi-θ-closed (resp., soft semi-θ-open) sets in $\tilde{X}$ is indicated by $\tilde{S}S\theta C(\tilde{X})$ (resp., $\tilde{S}S\theta O(\tilde{X})$).

Definition 1.8. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is known as:

(1) Soft extremally disconnected ($\tilde{S}ED$) [19], if $\tilde{s}cl(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$, $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$. Or $\tilde{s}int(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}^c$, $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

(2) Soft locally indiscrete ($\tilde{S}LI$) [20], if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}^c$, $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$. Or $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$, $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}^c$.

(3) Soft T_1-space [21], if $\tilde{e_x}, \tilde{e_y} \tilde{\in} \tilde{S}P(\tilde{X})$ such that $\tilde{e_x} \neq \tilde{e_y}$, there are $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{\tau}$ such that $\tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$, $\tilde{e_y} \tilde{\notin} (\mathcal{E}_1,\mathcal{P})$ and $\tilde{e_y} \tilde{\in} (\mathcal{E}_2,\mathcal{P})$, $\tilde{e_x} \tilde{\notin} (\mathcal{E}_2,\mathcal{P})$.

Proposition 1.9. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}ED$ iff

(1) $\tilde{S}RO(\tilde{X}) = \tilde{S}RC(\tilde{X})$ [22].

(2) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$ [1].

(3) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}\alpha O(\tilde{X})$ [1].

Proposition 1.10. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then,:

(1) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}\alpha O(\tilde{X})$ and $(\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$ [13].

(2) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\tau}$ and $(\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$ [23].

Lemma 1.11. [23] For any $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, we have:

(1) $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ iff $\tilde{s}cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(\mathcal{E}_1,\mathcal{P}))$.

(2) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ or $(\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then $\tilde{s}int(\tilde{s}cl((\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P}))) = \tilde{s}int(\tilde{s}cl(\mathcal{E}_1,\mathcal{P})) \tilde{\cap} \tilde{s}int(\tilde{s}cl(\mathcal{E}_2,\mathcal{P}))$.

Proposition 1.12. For any $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, the following statements hold:

(1) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$ iff $\tilde{s}int(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$ [24].

(2) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$ iff $\tilde{s}int(\mathcal{E},\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P})))$ [22].

(3) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$ iff $\tilde{s}scl(\mathcal{E},\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$ [22].

(4) $\tilde{s}s\theta cl(\mathcal{E},\mathcal{P}) = \tilde{s}scl(\mathcal{E},\mathcal{P})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. [18].

Proposition 1.13. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Then,:

(1) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ [1].

(2) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{Z} \tilde{\in} \tilde{S}\alpha O(\tilde{X})$ (resp., $\tilde{\tau}$, and $\tilde{S}S_pO(\tilde{X})$) [1].

(3) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ and $\tilde{Z} \tilde{\in} \tilde{S}RC(\tilde{X})$ (resp., $\tilde{Z} \tilde{\in} \tilde{S}CO(\tilde{X})$) [1].

(4) $\tilde{s}int(\mathcal{E},\mathcal{P}) = \tilde{s}int_{\tilde{Z}}(\mathcal{E},\mathcal{P})$, if $\tilde{Z} \tilde{\in} \tilde{\tau}$ [25].

Proposition 1.14. Let $(\tilde{Z},\tilde{\tau}_Z,\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$:

(1) If $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$) and $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ [1].

(2) $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$ and $\tilde{Z} \tilde{\in} \tilde{S}RC(\tilde{X})$ (resp. $\tilde{Z} \tilde{\in} \tilde{S}CO(\tilde{X})$). Then $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ [1].

(3) If $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ and $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{Z} \tilde{\cap} (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{Z})$ [26].

Theorem 1.15. For any $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, we have:

(1) (a) $\tilde{s}pcl(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cup} \tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))$,
(b) $\tilde{s}pint(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$ [8].

(2) (a) $\tilde{s}\alpha cl(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cup} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P})))$
(b) $\tilde{s}\alpha int(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}int(\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P})))$ [9].

(3) (a) $\tilde{s}scl(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cup} \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$,
(b) $\tilde{s}sint(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))$ [10].

(4) (a) $\tilde{s}bcl(\mathcal{E},\mathcal{P}) = \tilde{s}pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}scl(\mathcal{E},\mathcal{P})$,
(b) $\tilde{s}bint(\mathcal{E},\mathcal{P}) = \tilde{s}pint(\mathcal{E},\mathcal{P}) \tilde{\cup} \tilde{s}sint(\mathcal{E},\mathcal{P})$ [10].

(5) (a) $\tilde{s}\beta cl(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cup} \tilde{s}int(\tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P})))$
(b) $\tilde{s}\beta int(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P})))$ [24].

Proposition 1.16. [1] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}LI$ (resp., a soft T_1-space), then $\tilde{S}SO(\tilde{X}) = \tilde{S}S_cO(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

Corollary 1.17. [1] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}LI$, then:

(1) $\tilde{\tau} = \tilde{S}S_pO(\tilde{X})$.

(2) $\tilde{S}\alpha O(\tilde{X}) = \tilde{S}S_pO(\tilde{X})$.

(3) $\tilde{S}S_pO(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$.

Lemma 1.18. [1] Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$. Then,

(1) $\tilde{s}cl(\mathcal{E},\mathcal{P}) = \tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(2) $\tilde{s}cl(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$.

(3) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$.

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Lemma 1.19. [1] If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}ED$ and $\forall\,(\mathcal{E}_1,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$, then:
(1) $\tilde{s}cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}scl(\mathcal{E}_1,\mathcal{P})$.
(2) $\tilde{s}scl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}\alpha cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}pcl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}bcl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}\beta cl(\mathcal{E}_1,\mathcal{P})$.

Proposition 1.20. [1] Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}),(Q,\mathcal{P})\,\tilde{\subseteq}\,\tilde{X}$. If $(\mathcal{E},\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$ and $(Q,\mathcal{P})\,\tilde{\in}\,\tilde{S}CO(\tilde{X})$, then $(\mathcal{E},\mathcal{P})\,\tilde{\cap}\,(Q,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$.

Proposition 1.21. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{Z}\,\tilde{\in}\,\tilde{\tau}$. If $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}SC(\tilde{X})$, then $(C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}\,\tilde{\in}\,\tilde{S}SC(\tilde{Z})$.

Proof. Since $\tilde{Z}\,\tilde{\in}\,\tilde{\tau}$, then by Proposition 1.13(4),
$\tilde{s}int_{\tilde{Z}}(\mathcal{E},\mathcal{P}) = \tilde{s}int(\mathcal{E},\mathcal{P}),\ \forall\,(\mathcal{E},\mathcal{P})\,\tilde{\subseteq}\,\tilde{Z}$. Hence, we obtain $\tilde{s}int_{\tilde{Z}}(\tilde{s}cl_{\tilde{Z}}((C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}))$
$= \tilde{s}int(\tilde{s}cl((C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z})\,\tilde{\cap}\,\tilde{Z})$
$= \tilde{s}int(\tilde{s}cl((C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}))\,\tilde{\cap}\,\tilde{s}int\tilde{Z}\,\tilde{\subseteq}\,\tilde{s}int(\tilde{s}cl(C,\mathcal{P})\,\tilde{\cap}\,\tilde{s}cl(\tilde{Z}))\,\tilde{\cap}\,\tilde{Z}$
$= \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))\,\tilde{\cap}\,\tilde{s}int(\tilde{s}cl(\tilde{Z}))\,\tilde{\cap}\,\tilde{Z} = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))\,\tilde{\cap}\,\tilde{Z}$. Since $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}SC(\tilde{X})$, then $\tilde{s}int(\tilde{s}cl(C,\mathcal{P}))\,\tilde{\subseteq}\,(C,\mathcal{P})$. Thus,
$\tilde{s}int_{\tilde{Z}}(\tilde{s}cl_{\tilde{Z}}((C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}))\,\tilde{\subseteq}\,(C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}$. Therefore, $(C,\mathcal{P})\,\tilde{\cap}\,\tilde{Z}\,\tilde{\in}\,\tilde{S}SC(\tilde{Z})$.

2- Soft S_p-Closed Sets

Definition 2.1. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(C,\mathcal{P})\,\tilde{\subseteq}\,(\tilde{X},\tilde{\tau},\mathcal{P})$ is known as a soft S_p-closed set, if $\tilde{X}\,\tilde{\setminus}(C,\mathcal{P})$ is soft S_p-open. The family of all soft S_p-closed subsets of $(\tilde{X},\tilde{\tau},\mathcal{P})$ is indicated by $\tilde{S}S_pC(\tilde{X})$.

Remark 2.2. The definition indicates that $\tilde{S}S_pC(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}SC(\tilde{X})$. But, the converse is not true in general. The following examples illustrate the previous remark:

Example 2.3. Let $X = \{x_1,x_2\}$ and $\mathcal{P} = \{e_1,e_2\}$ with the soft topology $\tilde{\tau} = \{\tilde{\emptyset},\tilde{X},(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}),(\mathcal{E}_3,\mathcal{P}),(\mathcal{E}_4,\mathcal{P}),(\mathcal{E}_5,\mathcal{P}),(\mathcal{E}_6,\mathcal{P}),(\mathcal{E}_7,\mathcal{P})\}$ where $\tilde{\emptyset} = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(\mathcal{E}_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$, $(\mathcal{E}_2,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\emptyset)\}$, $(\mathcal{E}_3,\mathcal{P}) = \{(e_1,X),(e_2,\emptyset)\}$, $(\mathcal{E}_4,\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_2\})\}$, $(\mathcal{E}_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$, $(\mathcal{E}_6,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_2\})\}$ and $(\mathcal{E}_7,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a $\tilde{S}TS$ over X. The soft set $(\mathcal{E}_8,\mathcal{P}) = \{(e_1,\{x_2\}),\ (e_2,X)\}$ is soft semi-closed which is not soft S_p-closed.

Proposition 2.4. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P})\,\tilde{\subseteq}\,(\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$ iff $(C,\mathcal{P}) = \tilde{\bigcap}_{\vartheta\in\aleph}(D_\vartheta,\mathcal{P})$, where $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}SC(\tilde{X})$ and $(D_\vartheta,\mathcal{P})\,\tilde{\in}\,\tilde{S}PO(\tilde{X}),\ \forall\,\vartheta\,\in\,\aleph$.
Proof. Obvious.

Proposition 2.5. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$. Then,
(1) $\tilde{S}\theta C(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}S_pC(\tilde{X})$.
(2) $\tilde{S}S_cC(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}S_pC(\tilde{X})$.
(3) $\tilde{S}RO(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}S_pC(\tilde{X})$.
(4) $\tilde{S}CO(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}S_pC(\tilde{X})$.
Proof. Obvious.

Proposition 2.6. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}LI$ (resp., soft T_1-space), then $\tilde{S}SC(\tilde{X}) = \tilde{S}S_cC(\tilde{X}) = \tilde{S}S_pC(\tilde{X})$.
Proof. Applying Proposition 1.16 and Definition 2.1.

Corollary 2.7. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}LI$, then:
(1) $\tilde{S}S_pC(\tilde{X}) = \tilde{S}C(\tilde{X})$.
(2) $\tilde{S}S_pC(\tilde{X}) = \tilde{S}\alpha C(\tilde{X})$.
(3) $\tilde{S}S_pC(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}PC(\tilde{X})$.
Proof. Applying Corollary 1.17 and Definition 2.1.

Proposition 2.8. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}ED$ iff $\tilde{S}S_pC(\tilde{X})\,\tilde{\subseteq}\,\tilde{S}PC(\tilde{X})$ (resp., $\tilde{S}\alpha C(\tilde{X})$).

Proof. Let $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$. Then, $\tilde{X}\,\tilde{\setminus}(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$. Since $\tilde{X}$ is $\tilde{S}ED$, then by Proposition 1.9(2) (resp., Proposition 1.9(3)), $\tilde{X}\,\tilde{\setminus}(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}PO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$). Thus, $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}PC(\tilde{X})$ (resp., $\tilde{S}\alpha C(\tilde{X})$).
Conversely, let $(C,\mathcal{P})\,\tilde{\in}\,\tilde{\tau}^c$. Then, $\tilde{s}int(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$. That is, $\tilde{s}int(\mathcal{E},\mathcal{P})\,\tilde{\in}\,\tilde{S}RO(\tilde{X})$, so by Proposition 2.5(3), $\tilde{s}int(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$. By hypothesis, $\tilde{s}int(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}PC(\tilde{X})$ (resp., $\tilde{S}\alpha C(\tilde{X})$). That is, $\tilde{s}cl(\tilde{s}int(\tilde{s}int(C,\mathcal{P})))\,\tilde{\subseteq}\,\tilde{s}int(C,\mathcal{P})$ (resp., $\tilde{s}cl(\tilde{s}int(\tilde{s}cl(\tilde{s}int(C,\mathcal{P})))\,\tilde{\subseteq}\,\tilde{s}int(C,\mathcal{P})$), then $\tilde{s}cl(\tilde{s}int(C,\mathcal{P}))\,\tilde{\subseteq}\,\tilde{s}int(C,\mathcal{P})$, but $\tilde{s}int(C,\mathcal{P})\,\tilde{\subseteq}\,\tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$. Hence, $\tilde{s}int(C,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$. This means that, $\tilde{s}int(C,\mathcal{P})\,\tilde{\in}\,\tilde{\tau}^c$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}ED$.

Corollary 2.9. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$. Then, $\tilde{s}int(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$.

Proof. Since $(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$, then $\tilde{X}\,\tilde{\setminus}(C,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$. So, by Lemma 1.18(1), $\tilde{s}cl(\tilde{X}\,\tilde{\setminus}(C,\mathcal{P}))$ $= \tilde{s}cl(\tilde{s}int(\tilde{X}\,\tilde{\setminus}(C,\mathcal{P})))$ $\leftrightarrow$ $\tilde{X}\,\tilde{\setminus}(\tilde{s}int(C,\mathcal{P})) = \tilde{s}cl(\tilde{X}\,\tilde{\setminus}(\tilde{s}cl(C,\mathcal{P})))$ $\leftrightarrow$ $\tilde{X}\,\tilde{\setminus}(\tilde{s}int(C,\mathcal{P})) = \tilde{X}\,\tilde{\setminus}(\tilde{s}int(\tilde{s}cl\ (C,\mathcal{P})))$. Therefore, $\tilde{s}int(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$.

Lemma 2.10. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C_1,\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$. Then,:
(1) $\tilde{s}int(C_1,\mathcal{P}) = \tilde{s}pint(C_1,\mathcal{P})$.
(2) $\tilde{s}int(C_1,\mathcal{P}) = \tilde{s}\alpha int(C_1,\mathcal{P})$.
(3) $\tilde{s}sint(C_1,\mathcal{P}) = \tilde{s}bint(C_1,\mathcal{P}) = \tilde{s}\beta int(C_1,\mathcal{P})$.

Proof. (1) By of Theorem 1.15(1b) and Corollary 2.9, we have:
$\tilde{s}pint(C_1,\mathcal{P}) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(\tilde{s}cl(C_1,\mathcal{P})) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(C_1,\mathcal{P}) = \tilde{s}int(C_1,\mathcal{P})$.
(2) By of Theorem 1.15(2b) and Corollary 2.9, we have:
$\tilde{s}\alpha int(C_1,\mathcal{P}) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(\tilde{s}cl(\tilde{s}int(C_1,\mathcal{P}))) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(\tilde{s}cl(\tilde{s}int(\tilde{s}cl(C_1,\mathcal{P})))) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(\tilde{s}cl(C_1,\mathcal{P})) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}int(C_1,\mathcal{P}) = \tilde{s}int(C_1,\mathcal{P})$
(3) By Theorem 1.15(4b) and (1), we have:
$\tilde{s}bint(C_1,\mathcal{P}) = \tilde{s}pint(C_1,\mathcal{P})\,\tilde{\cup}\,\tilde{s}sint(C_1,\mathcal{P}) = \tilde{s}int(C_1,\mathcal{P})\,\tilde{\cup}\,\tilde{s}sint(C_1,\mathcal{P}) = \tilde{s}sint(C_1,\mathcal{P})$.
On the other hand, by Theorem 1.15(5b)(3b) and Corollary 2.9, we have:
$\tilde{s}\beta int(C_1,\mathcal{P}) = (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}cl(\tilde{s}int(\tilde{s}cl(C_1,\mathcal{P})))$
$= (C_1,\mathcal{P})\,\tilde{\cap}\,\tilde{s}cl(\tilde{s}int(C_1,\mathcal{P})) = \tilde{s}sint(C_1,\mathcal{P})$.

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Therefore, $\tilde{s}sint(C_1,\mathcal{P}) = \tilde{s}bint(C_1,\mathcal{P}) = \tilde{s}\beta int(C_1,\mathcal{P})$.

Proposition 2.11. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$ iff $\tilde{s}int(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$, then by Proposition 1.12(1), $\tilde{s}int(C,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. But by Proposition 2.5(3), we have $\tilde{S}RO(\tilde{X}) \tilde{\subseteq} \tilde{S}S_pC(\tilde{X})$, so $\tilde{s}int(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Conversely, let $\tilde{s}int(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Then Corollary 2.9, $\tilde{s}int(\tilde{s}int(C,\mathcal{P})) = \tilde{s}int(\tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$, so $\tilde{s}int(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$. Hence by Proposition 1.12(2), $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$.

Lemma 2.12. If $(\tilde{X},\tilde{\tau},\mathcal{P})$ is $\tilde{S}ED$ and $(C_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then:

(1) $\tilde{s}int(C_1,\mathcal{P}) = \tilde{s}sint(C_1,\mathcal{P})$.

(2) $\tilde{s}sint(C_1,\mathcal{P}) = \tilde{s}int(C_1,\mathcal{P}) = \tilde{s}\alpha int(C_1,\mathcal{P}) = \tilde{s}pint(C_1,\mathcal{P}) = \tilde{s}bint(C_1,\mathcal{P}) = \tilde{s}\beta int(C_1,\mathcal{P})$.

Proof. Applying Lemma 1.19 and Definition 2.1.

Proposition 2.13. For any $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, we have:

(1) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then $\tilde{s}pcl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(2) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}SC(\tilde{X})$, then $\tilde{s}pint(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(3) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{s}scl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(4) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $\tilde{s}sint(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(5) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}\alpha O(\tilde{X})$, then $\tilde{s}\beta\text{cl}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(6) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}\alpha C(\tilde{X})$, then $\tilde{s}\beta\text{int}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(7) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta O(\tilde{X})$, then $\tilde{s}\alpha\text{cl}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(8) If $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta C(\tilde{X})$, then $\tilde{s}\alpha\text{int}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. (1) By Theorem 1.15(1a), $\tilde{s}pcl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then $\tilde{s}pcl(C,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$. So, $\tilde{s}pcl(C,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$. By Lemma 1.18(3), thus $\tilde{s}pcl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(3) By Theorem 1.15(3a), $\tilde{s}scl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{s}scl(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$. So, $\tilde{s}scl(C,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. By Proposition 2.5(3), thus $\tilde{s}scl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(5) By Theorem 1.15(5a), $\tilde{s}\beta cl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}int(\tilde{s}cl(\tilde{s}int(C,\mathcal{P})))$. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}\alpha O(\tilde{X})$, then $\tilde{s}\beta cl(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\tilde{s}int(C,\mathcal{P})))$. So, $\tilde{s}\beta cl(C,\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X})$. By Proposition 2.5(3), thus $\tilde{s}\beta cl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

(7) By Theorem 1.15(2a), $\tilde{s}\alpha cl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}cl(\tilde{s}int(\tilde{s}cl(C,\mathcal{P})))$. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}\beta O(\tilde{X})$, then $\tilde{s}\alpha cl(C,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(\tilde{s}cl(C,\mathcal{P})))$. So, $\tilde{s}\alpha cl(C,\mathcal{P}) \tilde{\in} \tilde{S}RC(\tilde{X})$. By Lemma 1.18(3), thus $\tilde{s}\alpha cl(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

The remaining of this Proposition can be proven in the same way.

Theorem 2.14. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$. If $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, $(C,\mathcal{P}) \tilde{\subseteq} \tilde{Z}$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$, then $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

Proof. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then $(C,\mathcal{P}) \tilde{\in} \tilde{S}SC(\tilde{X})$ and $(C,\mathcal{P}) = \tilde{\cap}_{\vartheta \in \aleph} (D_\vartheta,\mathcal{P})$ where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, $\forall \vartheta \in \aleph$. Since $\tilde{Z} \tilde{\in} \tilde{\tau}$, then $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ and so by Proposition 1.14(3), $(D_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{Z})$, $\forall \vartheta \in \aleph$. Hence, $(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cap} \tilde{Z} = \tilde{\cap}_{\vartheta \in \aleph} (D_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z} = \tilde{\cap}_{\vartheta \in \aleph} ((D_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z})$. Therefore, by Proposition 2.4, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

Corollary 2.15. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(C,\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. If $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ and $\tilde{Z} \tilde{\in} \tilde{S}CO(\tilde{X})$, then $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

Proof. Since $\tilde{S}CO(\tilde{X}) \tilde{\subseteq} \tilde{\tau}$ and by Theorem 2.14, then $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

Proposition 2.16. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$). If $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

Proof. Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then $(C,\mathcal{P}) \tilde{\in} \tilde{S}SC(\tilde{X})$ and $(C,\mathcal{P}) = \tilde{\cap}_{\vartheta \in \aleph} (D_\vartheta,\mathcal{P})$ where $(D_\vartheta,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, $\forall \vartheta \in \aleph$. Then, $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} = (\tilde{\cap}_{\vartheta \in \aleph} (D_\vartheta,\mathcal{P})) \tilde{\cap} \tilde{Z} = \tilde{\cap}_{\vartheta \in \aleph} ((D_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z})$. Since $\tilde{Z} \tilde{\in} \tilde{\tau}$, by Proposition 1.21, $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}SC(\tilde{Z})$. Again, since $\tilde{Z} \tilde{\in} \tilde{\tau}$, then $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$, by Proposition 1.14(3), $(D_\vartheta,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}PO(\tilde{Z})$, $\forall \vartheta \in \aleph$. Then by Proposition 2.4, $(C,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pC(\tilde{Z})$.

3- On Soft S_p-Operators

In this section, the idea of soft S_p-open and soft S_p-closed sets is used to introduce and define several operators on soft topological spaces, such as soft S_p-neighborhood, soft S_p-derived, soft S_p-interior, soft S_p-closure, and soft S_p-boundary.

Definition 3.1. A $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(N,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ is known as a soft S_p-neighborhood of a soft subset $(\mathcal{E},\mathcal{P})$ of $\tilde{X}$, if there is $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (W,\mathcal{P}) \tilde{\subseteq} (N,\mathcal{P})$. If $(\mathcal{E},\mathcal{P}) = \tilde{e_x}$, then $(N,\mathcal{P})$ is known as a soft S_p-neighborhood of a soft point $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$.

The soft S_p-neighborhood system at $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$, indicated by $\tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$, is the family of all its soft S_p-neighborhood.

Proposition 3.2. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}), (\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then,

(1) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_2,\mathcal{P})$, and $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$, then $(\mathcal{E}_2,\mathcal{P}) \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$.

(2) $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$, $\forall \tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$.

(3) If $\{(\mathcal{E}_\lambda,\mathcal{P}): \lambda \in \Lambda\} \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$, then $\tilde{\cup} \{(\mathcal{E}_\lambda,\mathcal{P}): \lambda \in \Lambda\} \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$.

(4) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathrm{N}}_{S_p}(\tilde{e_x})$, then $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathrm{N}}_{s}(\tilde{e_x})$.

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Proof. (2) Let $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$. Then, $\tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})$. Therefore, $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathbb{N}}_{S_p}(\tilde{e_x})$.

Conversely, suppose that $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{\mathbb{N}}_{S_p}(\tilde{e_x})$, $\forall \tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$. Then, $\forall \tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$, there is $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})$. Therefore, $(\mathcal{E}_1,\mathcal{P}) = \tilde{\cup}\, \tilde{e_x} \tilde{\subseteq} \tilde{\cup}\, (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})$, $\forall \tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})$. It means that $(\mathcal{E}_1,\mathcal{P})$ is a soft union of soft S_p-open sets and hence, $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

The proof for the others is easy to do.

Remark 3.3. (1) In general, the opposite of part (4) of Proposition 3.2 is not always true.

(2) The soft intersection of two soft S_p-neighborhoods of a soft point need not be a soft S_p-neighborhood for that soft point.

As the next examples illustrates:

Example 3.4. In Example 2.3, (1) we have $(\mathcal{E}_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$ is a soft semi-neighborhood of $\widetilde{e_{1x_1}} = (e_1,\{x_1\})$, but it is not a soft S_p-neighborhood of $\widetilde{e_{1x_1}} = (e_1,\{x_1\})$.

(2) We have $(\mathcal{E}_9,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_1\})\}$ and $(\mathcal{E}_{10},\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_1\})\}$ are two soft S_p-neighborhood of $\widetilde{e_{2x_1}} = (e_2,\{x_1\})$, but $(\mathcal{E}_9,\mathcal{P}) \tilde{\cap} (\mathcal{E}_{10},\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_1\})\}$ is not soft S_p-neighborhood of $\widetilde{e_{2x_1}} = (e_2,\{x_1\})$.

Definition 3.5. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. A soft point $\tilde{e_x} \tilde{\in} \tilde{S}P(\tilde{X})$ is known as a soft S_p-limit point of $(\mathcal{E},\mathcal{P})$, if $(W,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$, $\forall (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e_x}$. The family of all soft S_p-limit points of $(\mathcal{E},\mathcal{P})$ is named soft S_p-derived set of $(\mathcal{E},\mathcal{P})$ and is indicated by $\tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

Proposition 3.6. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. If $(C,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$, $\forall (C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ containing $\tilde{e_x}$, then $\tilde{e_x} \tilde{\in} \tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

Proof. Let $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Then, there exists $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (C,\mathcal{P}) \tilde{\subseteq} (W,\mathcal{P})$. By assumption, we have $(C,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$, hence $(W,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$. Thus, $\tilde{e_x} \tilde{\in} \tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

Proposition 3.7. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$. Then, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ iff $\tilde{s}S_pD(C,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P})$.

Proof. Let $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ and $\tilde{e_x} \tilde{\in} \tilde{s}S_pD(C,\mathcal{P})$. On contrary, we suppose that $\tilde{e_x} \tilde{\notin} (C,\mathcal{P})$, then $\tilde{e_x} \tilde{\in} \tilde{X}\tilde{\setminus}(C,\mathcal{P})$ but since $\tilde{X}\tilde{\setminus}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{X}\tilde{\setminus}(C,\mathcal{P}) \tilde{\cap} ((C,\mathcal{P})\tilde{\setminus}\tilde{e_x}) \neq \tilde{\emptyset}$, which is a contradiction. Hence, $\tilde{e_x} \tilde{\in} (C,\mathcal{P})$. Thus, $\tilde{s}S_pD(C,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P})$.

Conversely, let $\tilde{s}S_pD(C,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P})$. To show $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$. Let $\tilde{e_x} \tilde{\in} \tilde{X}\tilde{\setminus}(C,\mathcal{P})$. Then, $\tilde{e_x} \tilde{\notin} (C,\mathcal{P})$, so $\tilde{e_x} \tilde{\notin} \tilde{s}S_pD(C,\mathcal{P})$, then there exists $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P})$ and $(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$. Therefore, $(W,\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(C,\mathcal{P})$. Thus, $\tilde{X}\tilde{\setminus}(C,\mathcal{P}) \tilde{\in} \tilde{\mathbb{N}}_{S_p}(\tilde{e_x})$ but since $\tilde{e_x}$ is arbitrary soft point of $\tilde{X}\tilde{\setminus}(C,\mathcal{P})$, so $\tilde{X}\tilde{\setminus}(C,\mathcal{P}) \tilde{\in} \tilde{\mathbb{N}}_{S_p}(\tilde{e_x}), \forall \tilde{e_x} \tilde{\in} \tilde{X}\tilde{\setminus}(C,\mathcal{P})$. By Proposition 3.2(2), $\tilde{X}\tilde{\setminus}(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Hence, $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

In the following result, several properties of the soft S_p-derived set are mentioned:

Proposition 3.8. For any $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, the following conditions hold:

(1) $\tilde{s}S_pD(\tilde{\emptyset}) = \tilde{\emptyset}$.

(2) $\tilde{e_x} \tilde{\in} \tilde{s}S_pD(\mathcal{E}_1,\mathcal{P})$ iff $\tilde{e_x} \tilde{\in} \tilde{s}S_pD((\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{e_x})$.

(3) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_2,\mathcal{P})$, then $\tilde{s}S_pD(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pD(\mathcal{E}_2,\mathcal{P})$.

(4) $\tilde{s}S_pD((\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_pD(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} \tilde{s}S_pD(\mathcal{E}_2,\mathcal{P})$

(5) $\tilde{s}S_pD(\mathcal{E}_1\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pD((\mathcal{E}_1,\mathcal{P}) \tilde{\cup} (\mathcal{E}_2,\mathcal{P}))$

(6) $\tilde{s}sD(\mathcal{E}_1\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pD(\mathcal{E}_1,\mathcal{P})$, where $\tilde{s}sD(\mathcal{E}_1,\mathcal{P})$ is a soft semi-derived set of $(\mathcal{E}_1,\mathcal{P})$.

Proof. Obvious.

In general, the opposite of parts (3), (4), (5) and (6) of Proposition 3.8 is not always true. As the next examples illustrates:

Example 3.9. In Example 2.3:

(1) Let $(\mathcal{E}_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ and $(\mathcal{E}_8,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,X)\}$. Then, $\tilde{s}S_pD(\mathcal{E}_5,\mathcal{P}) = \tilde{\emptyset}$ and $\tilde{s}S_pD(\mathcal{E}_8,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_2}},\widetilde{e_{2x_2}}\} = \{(e_1,X),(e_2,\{x_2\})\}$, so $\tilde{s}S_pD(\mathcal{E}_5,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pD(\mathcal{E}_8,\mathcal{P})$, but $(\mathcal{E}_5,\mathcal{P}) \tilde{\nsubseteq} (\mathcal{E}_8,\mathcal{P})$.

(2) Let $(\mathcal{E}_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ and $(\mathcal{E}_6,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_2\})\}$. Then, $\tilde{s}S_pD(\mathcal{E}_5,\mathcal{P}) = \tilde{\emptyset}$ and $\tilde{s}S_pD(\mathcal{E}_6,\mathcal{P}) = \tilde{\emptyset}$, and so $\tilde{s}S_pD(\mathcal{E}_5,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(\mathcal{E}_6,\mathcal{P}) = \tilde{\emptyset}$. But, $(\mathcal{E}_5,\mathcal{P}) \tilde{\cup} (\mathcal{E}_6,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$, so $\tilde{s}S_pD\big((\mathcal{E}_5,\mathcal{P}) \tilde{\cup} (\mathcal{E}_6,\mathcal{P})\big) = \{\widetilde{e_{2x_1}}\} = \{(e_2,\{x_1\})\}$.

Thus, $\tilde{s}S_pD((\mathcal{E}_5,\mathcal{P}) \tilde{\cup} (\mathcal{E}_6,\mathcal{P})) \tilde{\nsubseteq} \tilde{s}S_pD(\mathcal{E}_5,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(\mathcal{E}_6,\mathcal{P})$.

(3) Let $(\mathcal{E}_8,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,X)\}$, then $\tilde{s}sD(\mathcal{E}_8,\mathcal{P}) = \tilde{\emptyset}$ and $\tilde{s}S_pD(\mathcal{E}_8,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_2}},\widetilde{e_{2x_2}}\} = \{(e_1,X),(e_2,\{x_2\})\}$. Thus, $\tilde{s}S_pD(\mathcal{E}_8,\mathcal{P}) \tilde{\nsubseteq} \tilde{s}sD(\mathcal{E}_8,\mathcal{P})$.

Example 3.10. Let $X = \{x_1,x_2,x_3\}$ and $\mathcal{P} = \{e_1,e_2\}$ with the soft topology $\tilde{\tau} = \{\tilde{\emptyset},\tilde{X},(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}),(\mathcal{E}_3,\mathcal{P}),(\mathcal{E}_4,\mathcal{P}),(\mathcal{E}_5,\mathcal{P})\}$, where $\tilde{\emptyset} = \{(e_1,\emptyset),(e_2,\emptyset)\}$, $\tilde{X} = \{(e_1,X),(e_2,X)\}$, $(\mathcal{E}_1,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_1\})\}$, $(\mathcal{E}_2,\mathcal{P}) = \{(e_1,\{x_2,x_3\}),(e_2,\{x_1,x_3\})\}$ $(\mathcal{E}_3,\mathcal{P}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1,x_2\})\}$, $(\mathcal{E}_4,\mathcal{P}) = \{(e_1,X),(e_2,\{x_1,x_3\})\}$, $(\mathcal{E}_5,\mathcal{P}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_1\})\}$. Thus, $(\tilde{X},\tilde{\tau},\mathcal{P})$ is a $\tilde{S}TS$ over X. Let $(C_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_1,x_2\})\}$, and $(C,\mathcal{P}) = \{(e_1,\{x_1,x_2\}),(e_2,\{x_2\})\}$. Then, $\tilde{s}S_pD(C_1,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_2}},\widetilde{e_{1x_3}},\widetilde{e_{2x_2}},\widetilde{e_{2x_3}}\} = \{(e_1,X),(e_2,\{x_2,x_3\})\}$ and $\tilde{s}S_pD(C,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_3}},\widetilde{e_{2x_1}},\widetilde{e_{2x_2}},\widetilde{e_{2x_3}}\} = \{(e_1,\{x_1,x_3\}),(e_2,X)\}$, so $\tilde{s}S_pD(C_1,\mathcal{P}) \tilde{\cap} \tilde{s}S_pD(C,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_3}},\widetilde{e_{2x_2}},\widetilde{e_{2x_3}}\}$. But, $(C_1,\mathcal{P}) \tilde{\cap}$

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$(C,\mathcal{P})=\{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ so $\tilde{s}S_pD((C_1,\mathcal{P})\tilde{\cap}(C,\mathcal{P}))=\tilde{\emptyset}$. Thus, $\tilde{s}S_pD(C_1,\mathcal{P})\tilde{\cap}\tilde{s}S_pD(C,\mathcal{P})\tilde{\nsubseteq}\tilde{s}S_pD((C_1,\mathcal{P})\tilde{\cap}(C,\mathcal{P}))$.

Theorem 3.11. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. Then:

(1) $(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S_pC(\tilde{X})$.

(2) $\tilde{s}S_pD(\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\setminus}(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

(3) $\tilde{s}S_pD\left((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\right)\tilde{\subseteq}(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

Proof. (1) To prove $(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S_pC(\tilde{X})$. We shall prove that $\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\in}\tilde{S}S_pO(\tilde{X})$. Let $\tilde{e_x}\tilde{\in}\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))$. Then, $\tilde{e_x}\tilde{\notin}(\mathcal{E},\mathcal{P})$ and $\tilde{e_x}\tilde{\notin}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$, there exists $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})$ and $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$. Thus, $(W,\mathcal{P})\tilde{\subseteq}\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$. Also, if $(W,\mathcal{P})\tilde{\cap}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\neq\tilde{\emptyset}$, then there exists $\tilde{e_y}\tilde{\in}(W,\mathcal{P})\tilde{\cap}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$. This implies that $(W,\mathcal{P})\tilde{\cap}((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x})\neq\tilde{\emptyset}$, and so $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})\neq\tilde{\emptyset}$, which is a contradiction. So, $(W,\mathcal{P})\tilde{\cap}\tilde{s}S_pD(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$. Thus $(W,\mathcal{P})\tilde{\subseteq}\tilde{X}\tilde{\setminus}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$, so $(W,\mathcal{P})\tilde{\subseteq}\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))$. This means that $\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\in}\tilde{N}_{S_p}(\tilde{e_x})$ and since $\tilde{e_x}$ is arbitrary point of $\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))$. So, $\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\in}\tilde{N}_{S_p}(\tilde{e_x}),\forall\,\tilde{e_x}\tilde{\in}\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))$, then by Proposition 3.2(2), $\tilde{X}\tilde{\setminus}((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\in}\tilde{S}S_pO(\tilde{X})$. Hence, $(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S_pC(\tilde{X})$.

(2) Let $\tilde{e_x}\tilde{\in}\tilde{s}S_pD(\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\setminus}(\mathcal{E},\mathcal{P})$ and $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})$. Then, $(W,\mathcal{P})\tilde{\cap}(\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x})\neq\tilde{\emptyset}$. Let $\tilde{e_y}\tilde{\in}(W,\mathcal{P})\tilde{\cap}(\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x})$. Then, $\tilde{e_y}\tilde{\in}(W,\mathcal{P})$ and $\tilde{e_y}\tilde{\in}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$, so $(W,\mathcal{P})\tilde{\cap}((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_y})\neq\tilde{\emptyset}$, this means, there exists $\tilde{e_w}\tilde{\in}(W,\mathcal{P})\tilde{\cap}((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_y})$. Since $\tilde{e_w}\tilde{\in}(\mathcal{E},\mathcal{P})$, but $\tilde{e_x}\tilde{\notin}(\mathcal{E},\mathcal{P})$, so $\tilde{e_w}\neq\tilde{e_x}$. Hence, $(W,\mathcal{P})\tilde{\cap}((\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{e_x})\neq\tilde{\emptyset}$. Then, $\tilde{e_x}\tilde{\in}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

(3) By part (1), $(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{S}S_pC(\tilde{X})$. So by Proposition 3.7, we get $\tilde{s}S_pD((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P}))\tilde{\subseteq}(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_pD(\mathcal{E},\mathcal{P})$.

Definition 3.12. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. A soft point $\tilde{e_x}\tilde{\in}(\mathcal{E},\mathcal{P})$ is known as a soft S_p-interior point of $(\mathcal{E},\mathcal{P})$, if there exists $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$. The set of all soft S_p-interior points of $(\mathcal{E},\mathcal{P})$ is named a soft S_p-interior of $(\mathcal{E},\mathcal{P})$, and is indicated by $\tilde{s}S_pint(\mathcal{E},\mathcal{P})$. It is clear that $\tilde{s}S_pint(\mathcal{E},\mathcal{P})=\tilde{\cup}\{(W,\mathcal{P}):(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X}),(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})\}$.

Using Definition 3.1 and Definition 3.12, we can conclude the following result.

Corollary 3.13. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$, $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{e_x}\tilde{\in}\tilde{S}P(\tilde{X})$. Then, $(\mathcal{E},\mathcal{P})\tilde{\in}\tilde{N}_{S_p}(\tilde{e_x})$ iff $\tilde{e_x}\tilde{\in}\tilde{s}S_pint(\mathcal{E},\mathcal{P})$.

Proposition 3.14. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. If $\tilde{e_x}\tilde{\in}\tilde{s}S_pint(\mathcal{E},\mathcal{P})$, then there is $(C,\mathcal{P})\tilde{\in}\tilde{S}PC(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(C,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$.

Proof. Since $\tilde{e_x}\tilde{\in}\tilde{s}S_pint(\mathcal{E},\mathcal{P})$, then there exists $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$. Since $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$, so there is $(C,\mathcal{P})\tilde{\in}\tilde{S}PC(\tilde{X})$ containing $\tilde{e_x}$ such that $(C,\mathcal{P})\tilde{\subseteq}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$. Hence, $\tilde{e_x}\tilde{\in}(C,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$.

In the following result, several properties of the soft S_p-interior set are mentioned:

Proposition 3.15. For any $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$, the following conditions hold:

(1) $\tilde{s}S_pint(\tilde{\emptyset})=\tilde{\emptyset}$, $\tilde{s}S_pint(\tilde{X})=\tilde{X}$.

(2) $\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})$ is the largest soft S_p-open set contained in $(\mathcal{E}_1,\mathcal{P})$.

(3) $(\mathcal{E}_1,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ iff $(\mathcal{E}_1,\mathcal{P})=\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})$.

(4) $\tilde{s}S_pint(\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P}))=\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})$.

(5) If $(\mathcal{E}_1,\mathcal{P})\tilde{\subseteq}(\mathcal{E}_2,\mathcal{P})$, then $\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})\tilde{\subseteq}\tilde{s}S_pint(\mathcal{E}_2,\mathcal{P})$.

(6) $\tilde{s}S_pint((\mathcal{E}_1,\mathcal{P})\tilde{\cap}(\mathcal{E}_2,\mathcal{P}))\tilde{\subseteq}\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})\tilde{\cap}\tilde{s}S_pint(\mathcal{E}_2,\mathcal{P})$.

(7) $\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})\tilde{\cup}\tilde{s}S_pint(\mathcal{E}_2,\mathcal{P})\tilde{\subseteq}\tilde{s}S_pint((\mathcal{E}_1,\mathcal{P})\tilde{\cup}(\mathcal{E}_2,\mathcal{P}))$.

In general, $\tilde{\cup}_{\lambda\in\Lambda}\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})\tilde{\subseteq}\tilde{s}S_pint(\tilde{\cup}_{\lambda\in\Lambda}(\mathcal{E}_1,\mathcal{P}))$.

Proof. Obvious.

In general, the opposite of parts (5), (6) and (7) of Proposition 3.15 is not always true. As the next examples illustrates:

Example 3.16. In Example 3.10:

(1) Let $(B_1,\mathcal{P})=\{(e_1,\{x_2\}),(e_2,\{x_3\})\}$ and $(\mathcal{E}_1,\mathcal{P})=\{(e_1,\{x_2\}),(e_2,\{x_1\})\}$. Then, $\tilde{s}S_pint(B_1,\mathcal{P})=\tilde{\emptyset}$ and $\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})=(\mathcal{E}_1,\mathcal{P})$, so $\tilde{s}S_pint(B_1,\mathcal{P})\tilde{\subseteq}\tilde{s}S_pint(\mathcal{E}_1,\mathcal{P})$ but $(B_1,\mathcal{P})\tilde{\nsubseteq}(\mathcal{E}_1,\mathcal{P})$.

(2) Let $(B_2,\mathcal{P})=\{(e_1,\{x_2\}),(e_2,\{x_2\})\}$ and $(B_3,\mathcal{P})=\{(e_1,\{x_3\}),(e_2,\{x_1\})\}$. Then, $\tilde{s}S_pint(B_2,\mathcal{P})=\tilde{\emptyset}$, $\tilde{s}S_pcl(B_3,\mathcal{P})=\tilde{\emptyset}$, and so $\tilde{s}S_pint(B_2,\mathcal{P})\tilde{\cup}\tilde{s}S_pint(B_3,\mathcal{P})=\tilde{\emptyset}$. But, $(B_2,\mathcal{P})\tilde{\cup}(B_3,\mathcal{P})=(B_4,\mathcal{P})=\{(e_1,\{x_2,x_3\}),(e_2,\{x_1,x_2\})\}$, so $\tilde{s}S_pint(B_4,\mathcal{P})=(B_4,\mathcal{P})$. Thus, $\tilde{s}S_pint((B_2,\mathcal{P})\tilde{\cup}(B_3,\mathcal{P}))\tilde{\nsubseteq}\tilde{s}S_pint(B_2,\mathcal{P})\tilde{\cup}\tilde{s}S_pint(B_3,\mathcal{P})$.

Example 3.17. In Example 2.3, let $(\mathcal{E}_9,\mathcal{P})=\{(e_1,\{x_2\}),(e_2,\{x_1\})\}$ and $(\mathcal{E}_{10},\mathcal{P})=\{(e_1,\{x_1\}),(e_2,\{x_1\})\}$, then $\tilde{s}S_pint(\mathcal{E}_9,\mathcal{P})=(\mathcal{E}_9,\mathcal{P})$ and $\tilde{s}S_pint(\mathcal{E}_{10},\mathcal{P})=(\mathcal{E}_{10},\mathcal{P})$, so $\tilde{s}S_pint(\mathcal{E}_9,\mathcal{P})\tilde{\cap}\tilde{s}S_pint(\mathcal{E}_{10},\mathcal{P})=(\mathcal{E}_{14},\mathcal{P})=\{(e_1,\emptyset),(e_2,\{x_1\})\}$. But, $\tilde{s}S_pint\left((\mathcal{E}_9,\mathcal{P})\tilde{\cap}(\mathcal{E}_{10},\mathcal{P})\right)=\tilde{s}S_pint(\mathcal{E}_{14},\mathcal{P})=\tilde{\emptyset}$. Thus, $\tilde{s}S_pint(\mathcal{E}_9,\mathcal{P})\tilde{\cap}\tilde{s}S_pint(\mathcal{E}_{10},\mathcal{P})\tilde{\nsubseteq}\tilde{s}S_pint((\mathcal{E}_9,\mathcal{P})\tilde{\cap}(\mathcal{E}_{10},\mathcal{P}))$.

Proposition 3.18. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. Then:

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(1) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P}) = \tilde{\emptyset}$, then $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} \tilde{s}S_p int(\mathcal{E}_2,\mathcal{P}) = \tilde{\emptyset}$.

(2) $\tilde{s}S_p int((\mathcal{E}_1,\mathcal{P})\tilde{\setminus}(\mathcal{E}_2,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})\tilde{\setminus} \tilde{s}S_p int(\mathcal{E}_2,\mathcal{P})$.

(3) $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}sint(\mathcal{E}_1,\mathcal{P})$.

Proof. (1) Obvious.

(2) $\tilde{s}S_p int((\mathcal{E}_1,\mathcal{P})\tilde{\setminus}(\mathcal{E}_2,\mathcal{P})) = \tilde{s}S_p int((\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(\mathcal{E}_2,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} \tilde{s}S_p int(\tilde{X}\tilde{\setminus}(\mathcal{E}_2,\mathcal{P}))$ $\tilde{\subseteq} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E}_2,\mathcal{P})$.

(3) The proof arises from the fact that $\tilde{S}S_p O(\tilde{X}) \tilde{\subseteq} \tilde{S}SO(\tilde{X})$.

In general, the opposite of Proposition 3.18 is not always true. As the next examples illustrates:

Example 3.19. In Example 3.10, since $(B_5,\mathcal{P}) = \{(e_1,\{x_3\}),(e_2,\{x_1,x_2\})\}$ and $(B_6,\mathcal{P}) = \{(e_1,\{x_3\}), (e_2,\{x_1,x_3\})\}$, then $\tilde{s}S_p int(B_5,\mathcal{P}) = \tilde{\emptyset}$, $\tilde{s}S_p int(B_6,\mathcal{P}) = \tilde{\emptyset}$, and so $\tilde{s}S_p int(B_5,\mathcal{P}) \tilde{\cap} \tilde{s}S_p int (B_6,\mathcal{P}) = \tilde{\emptyset}$. But, $(B_5,\mathcal{P}) \tilde{\cap} (B_6,\mathcal{P}) \neq \tilde{\emptyset}$.

Example 3.20. In Example 2.3:

(1) Let $(\mathcal{E}_9,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_1\})\}$ and $(\mathcal{E}_{10},\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_1\})\}$. Then, $(\mathcal{E}_2,\mathcal{P}) = (\mathcal{E}_9,\mathcal{P})\tilde{\setminus}(\mathcal{E}_{10},\mathcal{P})$ and $\tilde{s}S_p int((\mathcal{E}_9,\mathcal{P})\tilde{\setminus}(\mathcal{E}_{10},\mathcal{P})) = \tilde{s}S_p int(\mathcal{E}_2,\mathcal{P}) = \tilde{\emptyset}$. But, $\tilde{s}S_p int(\mathcal{E}_9,\mathcal{P})\tilde{\setminus}\tilde{s}S_p int (\mathcal{E}_{10},\mathcal{P}) = (\mathcal{E}_2,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\emptyset)\}$. Thus, $\tilde{s}S_p int(\mathcal{E}_9,\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E}_{10},\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_p int((\mathcal{E}_9,\mathcal{P})\tilde{\setminus} (\mathcal{E}_{10},\mathcal{P}))$.

(2) We have $(\mathcal{E}_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\emptyset)\}$, then $\tilde{s}sint(\mathcal{E}_1,\mathcal{P}) = (\mathcal{E}_1,\mathcal{P})$ and $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) = \tilde{\emptyset}$. Thus, $\tilde{s}sint(\mathcal{E}_1,\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$.

Proposition 3.21. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then,

(1) $\tilde{s}S_p int(\tilde{s}sint(\mathcal{E}_1,\mathcal{P})) = \tilde{s}sint(\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$.

(2) $\tilde{s}S_p int(\tilde{s}S_c int(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_c int(\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_c int(\mathcal{E}_1,\mathcal{P})$.

Proof. (1) Since $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$, then $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. So, $\tilde{s}sint(\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$. By Proposition 3.18(3), we have $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}sint(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})$. We get $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_p int(\tilde{s}sint(\mathcal{E}_1,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_p int ((\mathcal{E}_1,\mathcal{P}))$, so $\tilde{s}S_p int(\tilde{s}Sint(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$.

(2) The proof is similar of part (1).

Theorem 3.22. Let $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) = (\mathcal{E}_1,\mathcal{P})\tilde{\setminus} \tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$.

Proof. Let $\tilde{e_x} \tilde{\in} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$. Then, $\tilde{e_x} \tilde{\notin} \tilde{s}S_p D (\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$. Since $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$, $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P})) = \tilde{\emptyset}$, then $\tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$, so $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$.

On the other hand, if $\tilde{e_x} \tilde{\in} (\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$, then $\tilde{e_x} \tilde{\notin} \tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$, so there exists $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$ containing $\tilde{e_x}$ such that $(W,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P})) = \tilde{\emptyset}$. That is, $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_1,\mathcal{P})$. Hence, $\tilde{e_x} \tilde{\in} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$. Thus, $(\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p D (\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P})) \tilde{\subseteq} \tilde{s}S_p int(\mathcal{E}_1,\mathcal{P})$. Therefore, $\tilde{s}S_p int(\mathcal{E}_1,\mathcal{P}) = (\mathcal{E}_1,\mathcal{P})\tilde{\setminus}\tilde{s}S_p D(\tilde{X}\tilde{\setminus}(\mathcal{E}_1,\mathcal{P}))$.

Definition 3.23. The soft intersection of all soft S_p-closed sets in $(\tilde{X},\tilde{\tau},\mathcal{P})$ containing $(C,\mathcal{P})$ is known as a soft S_p-closure of $(C,\mathcal{P})$ and is indicated by $\tilde{s}S_p cl(C,\mathcal{P})$, (i.e., $\tilde{s}S_p cl(C,\mathcal{P}) = \tilde{\cap}\{(D,\mathcal{P}):(D,\mathcal{P}) \tilde{\in} \tilde{S}S_p C(\tilde{X}),(C,\mathcal{P}) \tilde{\subseteq} (D,\mathcal{P})\}$).

The following result contains some properties of soft S_p-closure:

Proposition 3.24. For any $(\mathcal{E}_1,\mathcal{P})$, $(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, the following conditions hold.

(1) $\tilde{s}S_p cl(\tilde{\emptyset}) = \tilde{\emptyset}$, $\tilde{s}S_p cl(\tilde{X}) = \tilde{X}$.

(2) $\tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})$ is the smallest soft S_p-closed set containing $(\mathcal{E}_1,\mathcal{P})$.

(3) $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_p C(\tilde{X})$ iff $(\mathcal{E}_1,\mathcal{P}) = \tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})$.

(4) $\tilde{s}S_p cl(\tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})) = \tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})$.

(5) $\tilde{s}S_p D(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})$.

(6) If $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_2,\mathcal{P})$, then $\tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_p cl (\mathcal{E}_2,\mathcal{P})$.

(7) $\tilde{s}S_p cl\big((\mathcal{E}_1,\mathcal{P}) \tilde{\cap} (\mathcal{E}_2,\mathcal{P})\big) \tilde{\subseteq} \tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P}) \tilde{\cap} \tilde{s}S_p cl(\mathcal{E}_2,\mathcal{P})$.

(8) $\tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P}) \tilde{\cup} \tilde{s}S_p cl(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_p cl((\mathcal{E}_1,\mathcal{P}) \tilde{\cup} (\mathcal{E}_2,\mathcal{P}))$.

Proof. Obvious.

In general, the opposite of parts (5), (6), (7) and (8) of Proposition 3.24 is not always true. As the next examples illustrates:

Example 3.25. In Example 3.10:

(1) Let $(C_1,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_1,x_2\})\}$. Then, $\tilde{s}S_p D(C_1,\mathcal{P}) = \{\widetilde{e_{1x_1}},\widetilde{e_{1x_2}},\widetilde{e_{1x_3}},\widetilde{e_{2x_2}},\widetilde{e_{2x_3}}\} = \{(e_1,X),\ (e_2,\{x_2,x_3\})\}$ and $\tilde{s}S_p cl(C_1,\mathcal{P}) = \tilde{X}$. So, $\tilde{s}S_p cl(C_1,\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_p D(C_1,\mathcal{P})$.

(2) Let $(C_2,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ and $(\mathcal{E}_1,\mathcal{P}) = \{(e_1,\{x_2\}),(e_2,\{x_1\})\}$. Then, $\tilde{s}S_p cl(C_2,\mathcal{P}) = (C_2,\mathcal{P})$ and $\tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P}) = \tilde{X}$, so $\tilde{s}S_p cl(C_2,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_p cl(\mathcal{E}_1,\mathcal{P})$ but $(C_2,\mathcal{P}) \tilde{\nsubseteq} (\mathcal{E}_1,\mathcal{P})$.

(3) Let $(B_5,\mathcal{P}) = \{(e_1,\{x_3\}),(e_2,\{x_1,x_2\})\}$ and $(C_3,\mathcal{P}) = \{(e_1,\{x_2,x_3\}),(e_2,\{x_2\})\}$. Then, $\tilde{s}S_p cl(B_5,\mathcal{P}) = \tilde{X}$, $\tilde{s}S_p cl(C_3,\mathcal{P}) = \tilde{X}$, and so $\tilde{s}S_p cl(B_5,\mathcal{P}) \tilde{\cap} \tilde{s}S_p cl(C_3,\mathcal{P}) = \tilde{X}$. But, $(B_5,\mathcal{P}) \tilde{\cap} (C_3,\mathcal{P}) = (C_4,\mathcal{P}) = \{(e_1,\{x_3\}),(e_2,\{x_2\})\}$, so $\tilde{s}S_p cl(C_4,\mathcal{P}) = (C_4,\mathcal{P})$. Thus, $\tilde{s}S_p cl(B_5,\mathcal{P}) \tilde{\cap} \tilde{s}S_p cl(C_3,\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_p cl((B_5,\mathcal{P}) \tilde{\cap} (C_3,\mathcal{P}))$.

Example 3.26. In Example 2.3, we have $(\mathcal{E}_5,\mathcal{P}) = \{(e_1,\{x_1\}),(e_2,\{x_2\})\}$ and $(\mathcal{E}_6,\mathcal{P}) = \{(e_1,\{x_2\}), (e_2,\{x_2\})\}$, then $\tilde{s}S_p cl(\mathcal{E}_5,\mathcal{P}) = (\mathcal{E}_5,\mathcal{P})$ and $\tilde{s}S_p cl(\mathcal{E}_6,\mathcal{P}) = (\mathcal{E}_6,\mathcal{P})$. So, $\tilde{s}S_p cl(\mathcal{E}_5,\mathcal{P}) \tilde{\cup} \tilde{s}S_p cl (\mathcal{E}_6,\mathcal{P}) = (\mathcal{E}_7,\mathcal{P}) = \{(e_1,X),(e_2,\{x_2\})\}$ and $\tilde{s}S_p cl((\mathcal{E}_5,\mathcal{P}) \tilde{\cup} (\mathcal{E}_6,\mathcal{P})) = \tilde{s}S_p cl(\mathcal{E}_7,\mathcal{P}) = \tilde{X}$. Thus, $\tilde{s}S_p cl ((\mathcal{E}_5,\mathcal{P}) \tilde{\cup} (\mathcal{E}_6,\mathcal{P})) \tilde{\nsubseteq} \tilde{s}S_p cl(\mathcal{E}_5,\mathcal{P}) \tilde{\cup} \tilde{s}S_p cl(\mathcal{E}_6,\mathcal{P})$.

Proposition 3.27. Let $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{e_x} \tilde{\in} \tilde{s}S_p cl(C,\mathcal{P})$ iff $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_p O(\tilde{X})$ containing $\tilde{e_x}$, $(\mathcal{E},\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$.

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Proof. Let $\tilde{e}_x \tilde{\in} \tilde{s}S_pcl(C,\mathcal{P})$ and suppose that $(\mathcal{E},\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$, for some $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e}_x$. Then, $\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ and $(C,\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$, so by Proposition 3.24(2), $\tilde{s}S_pcl(C,\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$. This implies that $\tilde{e}_x \tilde{\in} \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$, which is contradiction. Therefore, $(\mathcal{E},\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$.

Conversely, let $(\mathcal{E},\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$, $\forall\, (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e}_x$. If $\tilde{e}_x \tilde{\notin} \tilde{s}S_pcl(C,\mathcal{P})$, then by Definition 3.23, there exists $(D,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ such that $(C,\mathcal{P}) \tilde{\subseteq} (D,\mathcal{P})$ but $\tilde{e}_x \tilde{\notin} (D,\mathcal{P})$. Thus, $\tilde{X}\tilde{\setminus}(D,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e}_x \tilde{\in} \tilde{X}\tilde{\setminus}(D,\mathcal{P})$ and therefore, $\tilde{X}\tilde{\setminus}(D,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$, which is a contradiction. Thus, $\tilde{e}_x \tilde{\in} \tilde{s}S_pcl(C,\mathcal{P})$.

Corollary 3.28. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ and $\tilde{e}_x \tilde{\in} \tilde{S}P(\tilde{X})$. If $(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$, $\forall\, (W,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\tilde{e}_x \tilde{\in} (W,\mathcal{P})$, then $\tilde{e}_x \tilde{\in} \tilde{s}S_pcl(C,\mathcal{P})$.

Proof. Let $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e}_x$. Then, there is $(W,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$ such that $\tilde{e}_x \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$. By hypothesis, $(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$ so, $(\mathcal{E},\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$. Therefore, by Proposition 3.27, $\tilde{e}_x \tilde{\in} \tilde{s}S_pcl(C,\mathcal{P})$.

Proposition 3.29. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{s}S_pcl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(C,\mathcal{P})$.

Proof. By Proposition 3.24(5), $\tilde{s}S_pD(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$ and $(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$, then $(C,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$.

On the other hand, by Proposition 3.24(2), $\tilde{s}S_pcl(C,\mathcal{P})$ is the smallest soft S_p-closed set containing $(C,\mathcal{P})$ and by Theorem 3.11(1), $(C,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, so $\tilde{s}S_pcl(C,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(C,\mathcal{P})$. Thus, $\tilde{s}S_pcl(C,\mathcal{P}) = (C,\mathcal{P}) \tilde{\cup} \tilde{s}S_pD(C,\mathcal{P})$.

Proposition 3.30. For any $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. The following statements are true:

(1) $\tilde{X}\tilde{\setminus}\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

(2) $\tilde{X}\tilde{\setminus}\tilde{s}S_pcl(C,\mathcal{P}) = \tilde{s}S_pint(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

(3) $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

(4) $\tilde{s}S_pcl(C,\mathcal{P}) = \tilde{X}\tilde{\setminus}\tilde{s}S_pint(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

Proof. (1) $\tilde{e}_x \tilde{\in} \tilde{X}\tilde{\setminus}\tilde{s}S_pint(C,\mathcal{P}) \leftrightarrow \tilde{e}_x \tilde{\notin} \tilde{s}S_pint(C,\mathcal{P}) \leftrightarrow \forall\, (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, with $\tilde{e}_x \tilde{\in} (W,\mathcal{P})$, $(W,\mathcal{P}) \tilde{\nsubseteq} (C,\mathcal{P}) \leftrightarrow$ By Proposition 3.27, $(W,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(C,\mathcal{P})) \neq \tilde{\emptyset}$, $\forall\, (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ with $\tilde{e}_x \tilde{\in} (W,\mathcal{P}) \leftrightarrow \tilde{e}_x \tilde{\in} \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

(2) $\tilde{e}_x \tilde{\in} \tilde{X}\tilde{\setminus}\tilde{s}S_pcl(C,\mathcal{P}) \leftrightarrow \tilde{e}_x \tilde{\notin} \tilde{s}S_pcl(C,\mathcal{P}) \leftrightarrow (W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$, $\exists\, (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e}_x \leftrightarrow \tilde{e}_x \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(C,\mathcal{P}) \leftrightarrow \tilde{e}_x \tilde{\in} \tilde{s}S_pint(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))$.

(3) By part (2), $\tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(C,\mathcal{P})) = \tilde{s}S_pint(\tilde{X}\tilde{\setminus}(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))) = \tilde{s}S_pint(C,\mathcal{P})$.

(4) By part (1), $\tilde{X}\tilde{\setminus}\tilde{s}S_pint(\tilde{X}\tilde{\setminus}(C,\mathcal{P})) = \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{X}\tilde{\setminus}(C,\mathcal{P}))) = \tilde{s}S_pcl(C,\mathcal{P})$.

Proposition 3.31. For any $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, we have $\tilde{s}scl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$.

Proof. Let $\tilde{e}_x \tilde{\in} \tilde{s}scl(C,\mathcal{P})$ and $\tilde{e}_x \tilde{\in} (W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$. Then, $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ and so $(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) \neq \tilde{\emptyset}$. By Proposition 3.27, $\tilde{e}_x \tilde{\in} \tilde{s}S_pcl(C,\mathcal{P})$. Thus, $\tilde{s}scl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$.

In general, the opposite of Proposition 3.31 is not always true. As the next examples illustrates:

Example 3.32. In Example 2.3, we have $(\mathcal{E}_{14},\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_1\})\}$, then $\tilde{s}scl(\mathcal{E}_{14},\mathcal{P}) = (\mathcal{E}_{14},\mathcal{P})$ and $\tilde{s}S_pcl(\mathcal{E}_{14},\mathcal{P}) = \tilde{X}$. Thus, $\tilde{s}S_pcl(\mathcal{E}_{14},\mathcal{P}) \tilde{\nsubseteq} \tilde{s}scl(\mathcal{E}_{14},\mathcal{P})$.

Proposition 3.33. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{s}\theta int(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_cint(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pint(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}sint(C,\mathcal{P}) \tilde{\subseteq} (C,\mathcal{P}) \tilde{\subseteq} \tilde{s}scl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_ccl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}\theta cl(C,\mathcal{P})$.

Proof. Obvious.

Proposition 3.34. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. If $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{s}s\theta cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}scl(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E}_1,\mathcal{P})$, where $\tilde{s}s\theta cl(\mathcal{E}_1,\mathcal{P})$ is soft semi-θ-closure of $(\mathcal{E}_1,\mathcal{P})$.

Proof. Since $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $(\mathcal{E}_1,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, so by Proposition 1.12(4), $\tilde{s}s\theta cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}scl(\mathcal{E}_1,\mathcal{P})$ and by Proposition 3.31, $\tilde{s}scl(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E}_1,\mathcal{P})$. Therefore, $\tilde{s}s\theta cl(\mathcal{E}_1,\mathcal{P}) = \tilde{s}scl(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E}_1,\mathcal{P})$.

Proposition 3.35. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(C,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. If $(C,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{s}S_pcl(C,\mathcal{P}) = \tilde{s}scl(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$.

Proof. By Proposition 3.31, we have $\tilde{s}scl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(C,\mathcal{P})$. So, it remains to prove that $\tilde{s}S_pcl(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}scl(C,\mathcal{P})$. Let $\tilde{e}_x \tilde{\notin} \tilde{s}scl(C,\mathcal{P})$ Then, there exists $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$ containing $\tilde{e}_x$ such that $(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$ and hence, $\tilde{s}cl(\tilde{s}int(W,\mathcal{P})) \tilde{\cap} \tilde{s}int(\tilde{s}cl(C,\mathcal{P})) = \tilde{\emptyset}$. Since $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$, then by Lemma 1.11(1), $\tilde{s}cl(W,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(W,\mathcal{P}))$ and $(C,\mathcal{P}) \tilde{\subseteq} \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$, so $\tilde{s}cl(W,\mathcal{P}) \tilde{\cap} (C,\mathcal{P}) = \tilde{\emptyset}$, by Lemma 1.18(2), $\tilde{s}cl(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e}_x$. Therefore, by Proposition 3.27, $\tilde{e}_x \tilde{\notin} \tilde{s}S_pcl(C,\mathcal{P})$. Thus, $\tilde{s}S_pcl(C,\mathcal{P}) = \tilde{s}scl(C,\mathcal{P})$.

For the second part, by Proposition 1.12(3), we have $\tilde{s}scl(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$. Hence, $\tilde{s}S_pcl(C,\mathcal{P}) = \tilde{s}scl(C,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(C,\mathcal{P}))$.

Corollary 3.36. If $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$, then $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}sint(C,\mathcal{P}) = \tilde{s}cl(\tilde{s}int(C,\mathcal{P}))$.

Proof. This follows from the use of soft complements and Propositions 3.35 and 3.30(2).

Proposition 3.37. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be $\tilde{S}ED$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{X}$. If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}RO(\tilde{X}) \cap \tilde{S}RC(\tilde{X})$.

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Proof. Since $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then by Proposition 1.9(2), $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Hence, by Proposition 3.35 and Proposition 1.9(1), we have $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P})) = \tilde{s}cl(\tilde{s}int(\mathcal{E},\mathcal{P}))$.

Proposition 3.38. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be $\tilde{S}ED$ and $(\mathcal{E},\mathcal{P}),(C,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then,:

(1) $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}scl(\mathcal{E},\mathcal{P}) = \tilde{s}cl(\mathcal{E},\mathcal{P})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(2) $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}sint(C,\mathcal{P}) = \tilde{s}int(C,\mathcal{P})$, if $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. (1) Since $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then by Proposition 1.9(2), $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Hence, by Proposition 3.35 and Lemma 1.19(1), we have $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}scl(\mathcal{E},\mathcal{P}) = \tilde{s}cl(\mathcal{E},\mathcal{P})$.

(2) Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then by Proposition 2.8, $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$. Hence, by Corollary 3.36 and Lemma 2.12(1), we have $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}sint(C,\mathcal{P}) = \tilde{s}int(C,\mathcal{P})$.

Proposition 3.39. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be $\tilde{S}ED$ and $(\mathcal{E},\mathcal{P}),(C,\mathcal{P}) \tilde{\subseteq} \tilde{X}$. Then:

(1) $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}scl(\mathcal{E},\mathcal{P}) = \tilde{s}cl(\mathcal{E},\mathcal{P}) = \tilde{s}bcl(\mathcal{E},\mathcal{P}) = \tilde{s}pcl(\mathcal{E},\mathcal{P}) = \tilde{s}\beta cl(\mathcal{E},\mathcal{P}) = \tilde{s}\alpha cl(\mathcal{E},\mathcal{P})$, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(2) $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}sint(C,\mathcal{P}) = \tilde{s}int(C,\mathcal{P}) = \tilde{s}bint(C,\mathcal{P}) = \tilde{s}pint(C,\mathcal{P}) = \tilde{s}\beta int(C,\mathcal{P}) = \tilde{s}\alpha int(C,\mathcal{P})$, if $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$.

Proof. (1) Since $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then by Proposition 1.9(2), $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Hence, by Proposition 3.35 and Lemma 1.19(2), $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}scl(\mathcal{E},\mathcal{P}) = \tilde{s}cl(\mathcal{E},\mathcal{P}) = \tilde{s}bcl(\mathcal{E},\mathcal{P}) = \tilde{s}pcl(\mathcal{E},\mathcal{P}) = \tilde{s}\beta cl(\mathcal{E},\mathcal{P}) = \tilde{s}\alpha cl(\mathcal{E},\mathcal{P})$.

(2) Since $(C,\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then by Proposition 2.8, $(C,\mathcal{P}) \tilde{\in} \tilde{S}PC(\tilde{X})$. Hence, by Corollary 3.36 and Lemma 2.12(2), $\tilde{s}S_pint(C,\mathcal{P}) = \tilde{s}sint(C,\mathcal{P}) = \tilde{s}int(C,\mathcal{P}) = \tilde{s}bint(C,\mathcal{P}) = \tilde{s}pint(C,\mathcal{P}) = \tilde{s}\beta int(C,\mathcal{P}) = \tilde{s}\alpha int(C,\mathcal{P})$.

Proposition 3.40. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$, $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}CO(\tilde{X})$ and $(W,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then,:

(1) $(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(W,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}))$.

(2) $\tilde{s}S_pint((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})) \tilde{\subseteq} (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pint(W,\mathcal{P})$.

Proof. (1) Let $\tilde{e_x} \tilde{\in} (\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(W,\mathcal{P})$. Then, $\tilde{e_x} \tilde{\in} (\mathcal{E},\mathcal{P})$ and $\tilde{e_x} \tilde{\in} \tilde{s}S_pcl(W,\mathcal{P})$. So, $\forall\, (G,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e_x}$, we have $(G,\mathcal{P}) \tilde{\cap} (W,\mathcal{P}) \neq \tilde{\emptyset}$. By Proposition 1.20, $(G,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{e_x} \tilde{\in} (G,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P})$. This implies that $((G,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P})) \tilde{\cap} (W,\mathcal{P}) \neq \tilde{\emptyset}$. Now, $(G,\mathcal{P}) \tilde{\cap} ((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})) \neq \tilde{\emptyset}$ and by Proposition 3.27, $\tilde{e_x} \tilde{\in} \tilde{s}S_pcl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}))$. Thus, $(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(W,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}))$.

(2) Part (1) and Proposition 3.30 provide the proof.

Proposition 3.41. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}),(W,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}\alpha O(\tilde{X})$ or $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$ and $(W,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$, then $\tilde{s}S_pcl\left((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})\right) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(W,\mathcal{P})$.

Proof. Let $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}\alpha O(\tilde{X})$ or $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{\tau}$ and $(W,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Then, by Proposition 1.10(1) or Proposition 1.10(2), $(\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. Since $\tilde{S}\alpha O(\tilde{X}) \tilde{\subseteq} \tilde{S}PO(\tilde{X})$ or $\tilde{\tau} \tilde{\subseteq} \tilde{S}PO(\tilde{X})$, then $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. So, $(\mathcal{E},\mathcal{P})$, $(W,\mathcal{P})$, and $(\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}) \tilde{\in} \tilde{S}PO(\tilde{X})$. By Proposition 3.35, $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P}))$, $\tilde{s}S_pcl(W,\mathcal{P}) = \tilde{s}int(\tilde{s}cl(W,\mathcal{P}))$, and $\tilde{s}S_pcl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})) = \tilde{s}int(\tilde{s}cl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})))$. Also, $\tilde{\tau} \tilde{\subseteq} \tilde{S}\alpha O(\tilde{X}) \tilde{\subseteq} \tilde{S}SO(\tilde{X})$, so $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. By Lemma 1.11(2), $\tilde{s}int(\tilde{s}cl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P}))) = \tilde{s}int(\tilde{s}cl(\mathcal{E},\mathcal{P})) \tilde{\cap} \tilde{s}int(\tilde{s}cl(W,\mathcal{P}))$. Hence by Proposition 3.35, $\tilde{s}S_pcl((\mathcal{E},\mathcal{P}) \tilde{\cap} (W,\mathcal{P})) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl(W,\mathcal{P})$.

Note: Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of a $\tilde{S}TS$ $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Then, $\tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$ and $\tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$ mean the soft S_p-closure and soft S_p-interior of $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$.

Now, we have the following results:

Proposition 3.42. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Then,:

(1) $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$, if $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$, $\tilde{\tau}$, $\tilde{S}S_pO(\tilde{X})$, $\tilde{S}CO(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$).

(2) $\tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E},\mathcal{P})$, if $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$).

Proof. (1) Let $\tilde{e_x} \tilde{\in} \tilde{s}S_pint(\mathcal{E},\mathcal{P})$. Then, there exists $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$. Since $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$, $\tilde{\tau}$, $\tilde{S}S_pO(\tilde{X})$, $\tilde{S}CO(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$), then by Proposition 1.13(1) (resp., Proposition 1.13(2), and Proposition 1.14(2)), $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{Z})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$. Therefore, $\tilde{e_x} \tilde{\in} \tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Thus, $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

(2) Let $\tilde{e_x} \tilde{\notin} \tilde{s}S_pcl(\mathcal{E},\mathcal{P})$. Then, there exists $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x} \tilde{\in} (W,\mathcal{P})$ and $(W,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. Since $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ and $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$), then by Proposition 1.14(1), $(W,\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\in} \tilde{S}S_pO(\tilde{Z})$. Now, if $\tilde{e_x} \tilde{\notin} \tilde{Z}$, then $\tilde{e_x} \tilde{\notin} \tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. If $\tilde{e_x} \tilde{\in} \tilde{Z}$, then $\tilde{e_x} \tilde{\in} (W,\mathcal{P}) \tilde{\cap} \tilde{Z}$ and we have $((W,\mathcal{P}) \tilde{\cap} \tilde{Z}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. Therefore, $\tilde{e_x} \tilde{\notin} \tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Thus, $\tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E},\mathcal{P})$.

Corollary 3.43. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{Z}$. Then,:

(1) $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\subseteq} \tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$, if $\tilde{Z} \tilde{\in} \tilde{S}SO(\tilde{X})$ (resp., $\tilde{S}\alpha O(\tilde{X})$, $\tilde{\tau}$, $\tilde{S}S_pO(\tilde{X})$, $\tilde{S}CO(\tilde{X})$, and $\tilde{S}RC(\tilde{X})$).

(2) $\tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{Z}$, if $\tilde{Z} \tilde{\in} \tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$).

Proof. (1) By Proposition 3.42(1), $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{Z} \tilde{\subseteq} \tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pint_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

(2) By Proposition 3.42(2), $\tilde{s}S_pcl_{\tilde{Z}}(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{Z}$.

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Proposition 3.44. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{Z}$. If $\tilde{Z}\tilde{\in}\tilde{S}RC(\tilde{X})$ (resp., $\tilde{S}CO(\tilde{X})$), then:
(1) $\tilde{s}S_p int_{\tilde{Z}}(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_p int(\mathcal{E},\mathcal{P})$.
(2) $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

Proof. (1) Let $\tilde{e_x}\tilde{\in}\tilde{s}S_p int_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Then, there exists $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{Z})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$. Since $\tilde{Z}\tilde{\in}\tilde{S}RC(\tilde{X})$ (resp., $\tilde{S}CO(\tilde{X})$), then by Proposition 1.13(3), $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$. Since $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\subseteq}(\mathcal{E},\mathcal{P})$, then $\tilde{e_x}\tilde{\in}\tilde{s}S_p int(\mathcal{E},\mathcal{P})$. Hence, $\tilde{s}S_p int_{\tilde{Z}}(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_p int(\mathcal{E},\mathcal{P})$.
(2) Let $\tilde{e_x}\tilde{\notin}\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Then, there exists $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{Z})$ containing $\tilde{e_x}$ such that $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$. Since $\tilde{Z}\tilde{\in}\tilde{S}RC(\tilde{X})$ (resp., $\tilde{S}CO(\tilde{X})$), then by Proposition 1.13(3), $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$. Since $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$, so $\tilde{e_x}\tilde{\notin}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})$. Thus, $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

Corollary 3.45. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{Z}$. If $\tilde{Z}\tilde{\in}\tilde{S}CO(\tilde{X})$, then:
(1) $\tilde{s}S_p int_{\tilde{Z}}(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})$.
(2) $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

Proof. This follows directly from Proposition 3.42 and Proposition 3.44.

Proposition 3.46. Let $(\tilde{Z},\tilde{\tau}_{\tilde{Z}},\mathcal{P})$ be a soft subspace of $(\tilde{X},\tilde{\tau},\mathcal{P})$. If $\tilde{Z}\tilde{\in}\tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$) and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{Z}$, then $\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{Z}$.

Proof. From Corollary 3.43(2), $\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})\tilde{\subseteq}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{Z}$. On the other hand, let $\tilde{e_x}\tilde{\in}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{Z}$. Then, $\tilde{e_x}\tilde{\in}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})$ and $\tilde{e_x}\tilde{\in}\tilde{Z}$. This is, for all $(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})$ and $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})\neq\tilde{\emptyset}$ and $\tilde{e_x}\tilde{\in}\tilde{Z}$. Since $\tilde{Z}\tilde{\in}\tilde{\tau}$ (resp., $\tilde{S}CO(\tilde{X})$), then by Proposition 1.14(1), $(W,\mathcal{P})\tilde{\cap}\tilde{Z}\tilde{\in}\tilde{S}S_pO(\tilde{Z})$ such that $\tilde{e_x}\tilde{\in}(W,\mathcal{P})\tilde{\cap}\tilde{Z}$ and $((W,\mathcal{P})\tilde{\cap}\tilde{Z})\tilde{\cap}(\mathcal{E},\mathcal{P})\neq\tilde{\emptyset}$. So, $\tilde{e_x}\tilde{\in}\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Hence, $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{Z}\tilde{\subseteq}\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$. Thus, $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{Z}=\tilde{s}S_p cl_{\tilde{Z}}(\mathcal{E},\mathcal{P})$.

Definition 3.47. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$. A soft point $\tilde{e_x}\tilde{\in}\tilde{S}P(\tilde{X})$ is known as a soft S_p-boundary point of $(\mathcal{E},\mathcal{P})$, if $\forall\,(W,\mathcal{P})\tilde{\in}\tilde{S}S_pO(\tilde{X})$ containing $\tilde{e_x}$, we have $(W,\mathcal{P})\tilde{\cap}(\mathcal{E},\mathcal{P})\neq\tilde{\emptyset}$ and $(W,\mathcal{P})\tilde{\cap}(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))\neq\tilde{\emptyset}$. Or equivalently, the soft S_p-boundary of $(\mathcal{E},\mathcal{P})$ is $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E},\mathcal{P})$ and the family of all soft S_p-boundary points of $(\mathcal{E},\mathcal{P})$ is indicated by $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.

Theorem 3.48. For any $(\mathcal{E},\mathcal{P})\tilde{\subseteq}(\tilde{X},\tilde{\tau},\mathcal{P})$, the following conditions hold.
(1) $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(2) $\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$.
(3) $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$.
(4) $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$ is a soft S_p-closed set.
(5) $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p\,Bd(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$.
(6) $\tilde{s}S_p Bd(\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))\tilde{\subseteq}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(7) $\tilde{s}S_p Bd(\tilde{s}S_p int(\mathcal{E},\mathcal{P}))\tilde{\subseteq}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(8) $\tilde{s}S_p Bd(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))\tilde{\subseteq}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(9) $\tilde{s}S_p int(\mathcal{E},\mathcal{P})=(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(10) $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})=(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$
(11) $\tilde{X}=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p int(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(12) $\tilde{X}\tilde{\setminus}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p int(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$.

Proof. (1) $\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}(\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E},\mathcal{P}))=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})$.
(2) $\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cap}(\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E},\mathcal{P}))=\tilde{\emptyset}$.
(3) $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{s}S_p int(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{X}\tilde{\setminus}\tilde{s}S_p int(\mathcal{E},\mathcal{P})$
$=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$ {by Proposition 3.30(1)}.
(4) $\tilde{s}S_p cl(\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))=\tilde{s}S_p cl(\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})))$
$\tilde{\subseteq}\tilde{s}S_p cl(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))\tilde{\cap}\tilde{s}S_p cl(\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})))$
$=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))=\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
Therefore, by Proposition 3.24(4), $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$ is a soft S_p-closed set.
(5) By part (3), $\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$
$=\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})))$
$=\tilde{s}S_p Bd(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))$ {by part (3)}.
(6) $\tilde{s}S_p Bd(\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))=\tilde{s}S_p cl(\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))\tilde{\cap}\tilde{s}S_p cl(\tilde{X}\tilde{\setminus}\,\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))$ {by part (3)}
$\tilde{\subseteq}\tilde{s}S_p cl(\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))=\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$ {by part (4)}.
(7) $\tilde{s}S_p Bd(\tilde{s}S_p int(\mathcal{E},\mathcal{P}))=\tilde{s}S_p cl(\tilde{s}S_p int(\mathcal{E},\mathcal{P}))\tilde{\setminus}\,\tilde{s}S_p int(\tilde{s}S_p int(\mathcal{E},\mathcal{P}))$
$=\tilde{s}S_p cl(\tilde{s}S_p int(\mathcal{E},\mathcal{P}))\tilde{\setminus}\,\tilde{s}S_p int(\mathcal{E},\mathcal{P})$ {by Proposition 3.15(5)}
$\tilde{\subseteq}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p int(\mathcal{E},\mathcal{P})=\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(8) $\tilde{s}S_p Bd(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))=\tilde{s}S_p cl(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))\tilde{\setminus}\,\tilde{s}S_p int(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))$
$=\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p int(\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))$ {by Proposition 3.24(5)}
$\tilde{\subseteq}\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p int(\mathcal{E},\mathcal{P})=\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.
(9) $(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})=(\mathcal{E},\mathcal{P})\tilde{\setminus}\,(\tilde{s}S_p cl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p int(\mathcal{E},\mathcal{P}))$
$=((\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p cl(\mathcal{E},\mathcal{P}))\tilde{\cup}((\mathcal{E},\mathcal{P})\tilde{\cap}\,\tilde{s}S_p int(\mathcal{E},\mathcal{P}))$
$=\tilde{\emptyset}\tilde{\cup}\tilde{s}S_p int(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})$.
(10) By part (1), $\tilde{s}S_p cl(\mathcal{E},\mathcal{P})=\tilde{s}S_p int(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$
$=((\mathcal{E},\mathcal{P})\tilde{\setminus}\,\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$ {by part (9)} $=\left((\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})\right)\tilde{\cap}(\tilde{X}\tilde{\setminus}\,\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P}))$
$=(\mathcal{E},\mathcal{P})\tilde{\cup}\tilde{s}S_p Bd(\mathcal{E},\mathcal{P})$.

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(11) This follows from (1) and Proposition 3.30(1).
(12) This follows from (11).

Remark 3.49. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E}_1,\mathcal{P}),(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ and $(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_2,\mathcal{P})$ does not imply that $\tilde{s}S_pBd(\mathcal{E}_1,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pBd(\mathcal{E}_2,\mathcal{P})$ or $\tilde{s}S_pBd(\mathcal{E}_2,\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pBd(\mathcal{E}_1,\mathcal{P})$, as the next example illustrates:

Example 2.50. In Example 2.3, we have $(\mathcal{E}_4,\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_2\})\}$, $(\mathcal{E}_{13},\mathcal{P}) = \{(e_1,\emptyset),(e_2,X)\} \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$ such that $(\mathcal{E}_4,\mathcal{P}) \tilde{\subseteq} (\mathcal{E}_{13},\mathcal{P})$. So, $\tilde{s}S_pBd(\mathcal{E}_4,\mathcal{P}) = (\mathcal{E}_4,\mathcal{P})$ and $\tilde{s}S_pBd(\mathcal{E}_{13},\mathcal{P}) = (\mathcal{E}_3,\mathcal{P})$, this show that $\tilde{s}S_pBd(\mathcal{E}_4,\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_pBd(\mathcal{E}_{13},\mathcal{P})$ and $\tilde{s}S_pBd(\mathcal{E}_{13},\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_pBd(\mathcal{E}_4,\mathcal{P})$.

Proposition 3.51. Let $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{s}sBd(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pBd(\mathcal{E},\mathcal{P})$.

Proof. Let $\tilde{e_x} \tilde{\in} \tilde{s}sBd(\mathcal{E},\mathcal{P})$ and $(W,\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ containing $\tilde{e_x}$. Then, $(W,\mathcal{P}) \tilde{\in} \tilde{S}SO(\tilde{X})$. Since $\tilde{e_x} \tilde{\in} \tilde{s}sBd(\mathcal{E},\mathcal{P})$, so $(W,\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) \neq \tilde{\emptyset}$ and $(W,\mathcal{P}) \tilde{\cap} (\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})) \neq \tilde{\emptyset}$. Hence, $\tilde{e_x} \tilde{\in} \tilde{s}S_pBd(\mathcal{E},\mathcal{P})$. Thus, $\tilde{s}sBd(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pBd(\mathcal{E},\mathcal{P})$.

In general, the opposite of Proposition 3.51 is not always true. As the next example illustrates:

Example 3.52. In Example 2.3, we have $(\mathcal{E}_{14},\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_1\})\}$, then $\tilde{s}sBd(\mathcal{E}_{14},\mathcal{P}) = (\mathcal{E}_{14},\mathcal{P})$ and $\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P}) = \tilde{X}$. Thus, $\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P}) \tilde{\nsubseteq} \tilde{s}sBd(\mathcal{E}_{14},\mathcal{P})$.

Proposition 3.53. For any $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$, the following conditions hold.

(1) If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$, then $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P})\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P})$.

(2) If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,(\mathcal{E},\mathcal{P})$.

(3) If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ and $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$, then $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P})$.

(4) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$ iff $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$ (i.e., $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$).

(5) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X})$ iff $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$.

(6) $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$ iff $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \tilde{\cap} \tilde{S}S_pC(\tilde{X})$.

Proof. The proof of (1)-(3) are obvious.

(4) Suppose that $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$, then by part(2), $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})\tilde{\setminus}\,(\mathcal{E},\mathcal{P})$. Hence, $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. That is $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})$.

Conversely, $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. Then, $\tilde{\emptyset} = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})\tilde{\setminus}\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{X}\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P})$. Since $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{s}S_pcl(\mathcal{E},\mathcal{P})$, then $\tilde{X}\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\cap} (\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. Thus, $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} \tilde{X}\tilde{\setminus}(\tilde{X}\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P})) = \tilde{s}S_pint(\mathcal{E},\mathcal{P})$. But always $\tilde{s}S_pint(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$. This implies that $(\mathcal{E},\mathcal{P}) = S_pint(\mathcal{E},\mathcal{P})$. Therefore, $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X})$.

(5) $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pC(\tilde{X}) \leftrightarrow \tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \leftrightarrow$ by part (4), $\tilde{s}S_pBd(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})) \tilde{\subseteq} (\mathcal{E},\mathcal{P}) \leftrightarrow$ by Theorem 3.48(5), $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{s}S_pBd(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})) \tilde{\subseteq} (\mathcal{E},\mathcal{P})$.

(6) Suppose that $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$, then $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$. This means that $\tilde{s}S_pcl(\mathcal{E},\mathcal{P}) = \tilde{s}S_pint(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P})$. Hence, $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \tilde{\cap} \tilde{S}S_pC(\tilde{X})$.

Conversely, if $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \tilde{\cap} \tilde{S}S_pC(\tilde{X})$, then $(\mathcal{E},\mathcal{P}) = \tilde{s}S_pint(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})$. Hence, $\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P})\tilde{\setminus}\, \tilde{s}S_pint(\mathcal{E},\mathcal{P}) = (\mathcal{E},\mathcal{P})\tilde{\setminus}(\mathcal{E},\mathcal{P}) = \tilde{\emptyset}$.

Proposition 3.54. Let $(\tilde{X},\tilde{\tau},\mathcal{P})$ be a $\tilde{S}TS$ and $(\mathcal{E},\mathcal{P}) \tilde{\subseteq} (\tilde{X},\tilde{\tau},\mathcal{P})$. Then, $\tilde{s}S_pBd(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))) = \tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$.

Proof. $\tilde{s}S_pBd\left(\tilde{s}S_pBd\left(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})\right)\right) = \tilde{s}S_pcl\left(\tilde{s}S_pBd\left(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})\right)\right)$
$\tilde{\cap}\, \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))))$
$=\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) \tilde{\cap}\, \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))))$
{by Theorem 3.48(3) and Proposition 3.24(3)}. …….(1)

Now, we have
$\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})))$
$= \tilde{X}\tilde{\setminus}[\tilde{s}S_pcl(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) \tilde{\cap}\, \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})))]$
$= \tilde{X}\tilde{\setminus}[\tilde{s}S_pBd(\mathcal{E},\mathcal{P}) \tilde{\cap}\, \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})))]$ {by Theorem 3.48(10)}
$= (\tilde{X}\tilde{\setminus}\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) \tilde{\cup} (\tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))))$.

Therefore, by Proposition 3.24(8), we obtain:
$\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) =$
$\tilde{s}S_pcl[(\tilde{X}\tilde{\setminus}\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$
$\tilde{\cup}\, (\tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))))]$
$\tilde{\supseteq}\, \tilde{s}S_pcl\left[\left(\tilde{X}\tilde{\setminus}\tilde{s}S_pBd(\mathcal{E},\mathcal{P})\right)\right]$
$\tilde{\cup}\, \tilde{s}S_pcl[(\tilde{X}\tilde{\setminus}\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))))]$
$= (W,\mathcal{P}) \tilde{\cup}\, \tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(W,\mathcal{P})) = \tilde{X}$, where $(W,\mathcal{P}) = \tilde{s}S_pcl[(\tilde{X}\tilde{\setminus}\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))]$. So,
$\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})))) = \tilde{X}$. ......(2)

From (1) and (2), we obtain:
$\tilde{s}S_pBd(\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))) =$
$\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) \tilde{\cap} \tilde{X} = \tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$.

Now, we will show by an example that the opposite of part (6) of Theorem 3.48 is not always true in general. Thus, $\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) \neq \tilde{s}S_pBd(\mathcal{E},\mathcal{P})$:

Example 3.55. In Example 2.3, we have $(\mathcal{E}_{14},\mathcal{P}) = \{(e_1,\emptyset),(e_2,\{x_1\})\}$, then $\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P}) = \tilde{X}$ and $\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P})) = \tilde{\emptyset}$. Thus, $\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P}) \tilde{\nsubseteq} \tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P}))$ and hence, $\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P})) \neq \tilde{s}S_pBd(\mathcal{E}_{14},\mathcal{P})$.

However we have the following result:

Proposition 3.56. If $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \tilde{\cup} \tilde{S}S_pC(\tilde{X})$, then $\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) = \tilde{s}S_pBd(\mathcal{E},\mathcal{P})$.

Proof. Since $(\mathcal{E},\mathcal{P}) \tilde{\in} \tilde{S}S_pO(\tilde{X}) \tilde{\cup} \tilde{S}S_pC(\tilde{X})$, then $\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P})) = \tilde{s}S_pcl(\mathcal{E},\mathcal{P}) \tilde{\cap} \tilde{s}S_pcl$

$(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))\,\tilde{\cap}\,\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}\,\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$. Since $(\mathcal{E},\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pO(\tilde{X})$ (resp., $(\mathcal{E},\mathcal{P})\,\tilde{\in}\,\tilde{S}S_pC(\tilde{X})$), then by Proposition 3.53(4) (resp., Proposition 3.53(5)), $\tilde{s}S_pBd(\mathcal{E},\mathcal{P})\,\tilde{\cap}\,(\mathcal{E},\mathcal{P})=\tilde{\emptyset}$ (resp., $\tilde{s}S_pBd(\mathcal{E},\mathcal{P})\,\tilde{\subseteq}\,(\mathcal{E},\mathcal{P})$). This implies that $(\mathcal{E},\mathcal{P})\,\tilde{\subseteq}\,\tilde{X}\tilde{\setminus}\,\tilde{s}S_pBd(\mathcal{E},\mathcal{P})$ (resp., $\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P})\,\tilde{\subseteq}\,\tilde{X}\tilde{\setminus}\,\tilde{s}S_pBd(\mathcal{E},\mathcal{P})$). Hence, $\tilde{s}S_pcl(\mathcal{E},\mathcal{P})\,\tilde{\subseteq}\,\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}\,\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$ (resp., $\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))\,\tilde{\subseteq}\,\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}\,\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))$). Thus,:
$\tilde{s}S_pBd(\tilde{s}S_pBd(\mathcal{E},\mathcal{P}))=\tilde{s}S_pcl(\mathcal{E},\mathcal{P})\,\tilde{\cap}\,\tilde{s}S_pcl(\tilde{X}\tilde{\setminus}(\mathcal{E},\mathcal{P}))=\tilde{s}S_pBd(\mathcal{E},\mathcal{P})$.

4- Conclusion

In this paper, we introduce soft S_p-closed sets as a new class of soft sets which is contained in the class of soft semi-closed sets. Also, via soft S_p-closedness and soft S_p-openness, several new soft topological operators are defined. Several characterizations, properties, relationships, and examples regarding the new concepts are introduced. These results help us in the future to study new classes of soft functions such as soft almost S_p-continuity and soft weakly S_p-continuity.

References

[1] P. M. Mahmood, H. M. Darwesh, H. A. Shareef, and S. Al Ghour, "A Stronger Novel Form of Soft Semi-Open Set," *New Mathematics and Natural Computation, December 2023*, doi: https://doi.org/10.1142/S1793005725500164.
[2] D. Molodtsov, "Soft Set Theory- First Results," *Computers and Mathematics with Applications*, vol. 37, no. 4–5, pp. 19–31, 1999.
[3] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers and Mathematics with Applications*, vol. 45, no. 4–5, pp. 555–562, 2003.
[4] S. Das and S. K. Samanta, "Soft metric," *Annals of Fuzzy Mathematics and Informatics*, vol. 6, no. 1, pp. 77–94, 2013.
[5] M. Shabir and M. Naz, "On soft topological spaces," *Computers and Mathematics with Applications*, vol. 61, no. 7, pp. 1786–1799, 2011.
[6] S. Hussain and B. Ahmad, "Some properties of soft topological spaces," *Computers and Mathematics with Applications*, vol. 62, no. 11, pp. 4058–4067, 2011.
[7] J. Mahanta and P. K. Das, "On soft topological space via semiopen and semiclosed soft sets," *Kyungpook Mathematical Journal*, vol. 54, no. 2, pp. 221–236, 2014.
[8] G. Ilango and M. Ravindran, "On Soft Preopen Sets in Soft Topological Spaces," *International Journal of Mathematics Research*, vol. 5, no. 4, pp. 399–409, 2013.
[9] M. Akdag and A. Ozkan, "Soft α -Open Sets and Soft α -Continuous Functions," *Abstract and Applied Analysis*, vol. 2014, Art., no. 1, p. 7, 2014.
[10] M. Akdag and A. Ozkan, "Soft b-open sets and soft b-continuous functions," *Mathematical Sciences*, vol. 8, no. 2, Sep. 2014.
[11] I. Arockiarani and A. A. Lancy, "Generalized soft gβ closed sets and soft gsβ closed sets in soft topological spaces," *International Journal of Mathematical Archive*, vol. 4, no. 2, pp. 17–23, 2013.
[12] S. Yüksel, N. Tozlu, and Z. G. Ergül, "Soft regular generalized closed sets in soft topological spaces," *International Journal of Mathematical Analysis*, vol. 8, no. 5–6, pp. 355–367, 2014.
[13] S. Y. Khalaf, A. B., & Musa, "SSc-open Sets in Soft Topological Spaces," *Journal of Garmian University*, vol. 1, pp. 1–22, 2015.
[14] B. Chen, "Soft Semi-open sets and related properties in soft topological spaces," *Applied Mathematics and Information Sciences*, vol. 7, no. 1, pp. 287–294, 2013.
[15] M. Akdag and A. Ozkan, "On Soft β -open sets and soft β -continuous functions," *Scientific World Journal*, vol. 2014, no. Article ID 843456, p. 6, 2014.
[16] S. Hussain, "Properties of Soft Semi-open and Soft semi-closed Sets Properties," *Pensee Journal*, vol. 76(2), pp. 133–143, 2014.
[17] D. N. Georgiou, A. C. Megaritis, and V. I. Petropoulos, "On soft topological spaces," *Applied Mathematics and Information Sciences*, vol. 7, no. 5, pp. 1889–1901, 2013.
[18] S. Hussain, "On soft regular-open (closed) sets in soft topological spaces," *Journal of applied mathematics & informatics*, vol. 36, no. 1–2, pp. 59–68, 2018.
[19] A. Aclkgoz and Nihal A. Tas, "Some New Soft Sets and Decompositions of Some Soft Continuities," *Annals of Fuzzy Mathematics and Informatics*, vol. 9, no. 1, pp. 23–35, 2015.
[20] R. A. Hosny and D. Al-Kadi, "Soft semi open sets with respect to soft ideals," *Applied Mathematical Sciences*, vol. 8, no. 149–152, pp. 7487–7501, 2014.
[21] S. Hussain and B. Ahmad, "Soft separation axioms in soft topological spaces," *Hacettepe Journal of Mathematics and Statistics*, vol. 44, no. 3, pp. 559–568, 2015.
[22] B. A. Asaad, "Results on soft extremally disconnectedness of soft topological spaces," *Journal of Mathematics and Computer Science*, vol. 17, no. 04, pp. 448–464, 2017.
[23] A. Kandil, O. El-tantawy, S. A. El-Sheikh, and A. M. A. El-latif, "γ-operation and decompositions of some forms of soft continuity in soft topological spaces," *Annals of Fuzzy Mathematics and Informatics*, vol. 7, no. 2, pp. 181–196, 2014.
[24] Y. Yumak and A. K. Kaymakcı, "Soft β -open sets and their applications," *Journal of New Theory*, no. 4, pp. 80–89, 2015.
[25] T. Aydın and S. Enginoğlu, "Some results on soft topological notions," *Journal of New Results in*

Science, vol. 10, no. 1, pp. 65–75, 2021.
[26] M. E. Al-shami, T. M., & El-Shafei, "On soft compact and soft Lindelof spaces via soft pre-open sets," *Annals of Fuzzy Mathematics and Informatics*, vol. 17, no. 1, pp. 79–100, 2019.