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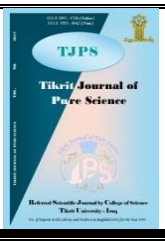

Dynamical symmetry of $^{102}_{42}Mo$ by utilizing (IBM-1)

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ABSTRACT

The structure of ($^{102}_{42}$Mo) was explored theoretically with IBM-1 in this work. The $\frac{E4_1^+}{E2_1^+}$ symmetry limit of the proposed nucleus was investigated. The considered nucleus was discovered to be deformed with O(6) dynamical symmetry area with gamma-ray softened nucleus. In addition, the B(E2) transition, quadrupole moment, and energy levels for all bands would be computed in this study. According to recent research, ($^{102}_{42}$Mo) is unstable to gamma, and their transition behaviors are generally similar to symmetric (γ -soft) O(6) limits. When the current findings are compared to the experimental results, they show a high level of agreement.

التماثل الديناميكي لنظير $^{102}_{42}Mo$ باستخدام (IBM-1)

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الملخص

تم استكشاف بنية $^{102}_{42}Mo$ نظريًا باستخدام IBM-1 في هذا العمل. تم دراسة حد التناظر ($\frac{E4_1^+}{E2_1^+}$) للنواة المقترحة واكتشاف أن النواة المدروسة مشوهة بمنطقة التناظر الديناميكي O(6) مع نواة مخففة لأشعة جاما. بالإضافة إلى ذلك، تم حساب الانتقال B(E2) والعزم الرباعي ومستويات الطاقة لجميع النطاقات في هذه الدراسة. وفقًا لأبحاث حديثة، $^{102}_{42}Mo$ غير مستقر لجاما، وسلوكياتها الانتقالية قريبة من حدود O(6) المتماثلة (γ-soft). عندما تتم مقارنة النتائج الحالية مع النتائج التجريبية، فإنها تظهر مستوى عال من الاتفاق.

1. Introduction

The Interacting Boson Model (IBM) was put together by (Arima and Iachello) to look into the structure and properties of even-even nuclei, as well as, characterize communal excitations in nuclei of atoms. The use of the strategy restrictions of the boson operators enables analytic interpretations of the configurations and anticipation values for the three separate optimal bounds of nuclei. The overall quantity of bosons in IBM-1 is dictated by how many couples involve protons and neutrons that exist beyond their enclosed shells, with no distinction established among proton-type and neutron-type

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bosons [1]. IBM-1 corresponds to a (s-d) boson system featuring six components which are capable of being compared to six-dimensional space. According to group theory, which will end up in a description in terms of U(6). The nucleon or hole pairs in the IBM-1 must belong to the same sort of nucleon. Indicating that proton and neutron couples haven't been mentioned. In the context of second quantization, nuclear states are characterized [2]. The unitary group of six dimensions contains three more subgroups, which are depicted in the scheme diagram below.

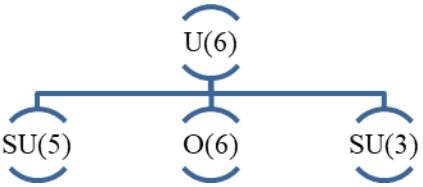

Fig. 1: Classification scheme for the unitary group in six dimensions

As already mentioned, the numerous variants of U(6) create several dynamical geometries, that are relevant concerning the vibrator, deformed rotor, and (γ-soft) axial order [3]. Dynamical symmetry is important in nuclear structure and is most clearly recognized through the interacting boson model. Every one of these circumstances was actually observed in reality, and every cluster was defined by a self-quantum number, and the ratio of energy levels in each nucleus within that group was determined through which quantum number [4]. In the shell known as valence, these bosons are identified as interconnected pairs of protons linked by pairs of neutrons. As a result, it restricts the boson number, as is calculated by computing the total number of particle pairs (separately for protons and neutrons) if the outermost protective shell is less than half filled. Determine the total number of hole pairs while the outer layer is over fifty percent full [5-7]. The present work tries to assess $^{102}_{42}Mo$'s nuclear structure such as (energy levels, B(E2) transition, dynamical symmetry, nuclear shape, and quadrupole moment value) considering the constraints of the broader (IBM-1) paradigm.

2. The IBM-1 origins

The Interacting Boson Model (IBM-1) is a theoretical model designed for illustrating atomic nuclei's interactions. It assumes that atomic nuclei may be seen as assemblies of interacting bosons, which symbolize pairs of nucleons (protons and neutrons) that are linked to one another in the core of the atom. The Hamiltonian in the Interacting Boson Model-1 defines the interactions among couples of s-bosons with pairs of d-bosons inside the atomic nucleus. The IBM-1 program is based on a simple Hamiltonian with six parameters that could have been derived from experimental data. Such the Hamiltonian operator ($\hat{H}$) contains one and two body operators [1,2].

$\hat{H} = \varepsilon_s s^\dagger \tilde{s} + \varepsilon_d \sum_m d^\dagger \tilde{d} + V$ …..(1)

Where ε_s and ε_d are s and d single – boson energies, V is boson-boson interaction potential, $s^\dagger$ ($\tilde{s}$) are creation and annihilation operators for the s-state, (s-boson), and $d^\dagger$($\tilde{d}$) are creation (annihilation) operators for the d-state, (d-boson). IBM-1 Hamiltonian is indicated to be [1-5]:

$\hat{H} = \varepsilon\ (n_d) + a_0\ (\hat{P}.\hat{P}) + a_1(\hat{L}.\hat{L}) + a_2(\hat{Q}.\hat{Q}) + a_3\ (\hat{T}_3.\hat{T}_3) + a_4\ (\hat{T}_4.\hat{T}_4)$ ….(2)

E2 transitions constitute among the more essential electromagnetic properties. E2 algorithm was used to compute the B(E2) values. The total amount of bosons should be preserved because the E2 transition operator (electric quadrupole transition operator) would be the Hermitian tensor of rank two. Because the universal E2 activator may be expressed with this constraint:

$T(E2) = \alpha_B Q$ …. (3)

Where α_B, Q denotes to the effective boson charge and quadrupole moment operator respectively [5-10].

3. Results and Discussion

The energy levels for $^{102}_{42}Mo$ was demonstrated through the addition of the settings mentioned in tables (1) to IBM-1's Hamiltonian equations, and IBM-1 obtained low-lying levels of energy that fall beneath the O(6) limit. Employing the computer software program PHINT [11], the IBM-1 Hamiltonian was created and calculated according to an O(6) premise. The value of the parameters is autonomous quantities that were meticulously designed to nearly exactly mimic every positive parity level's energy of excitement.

Table 1: Hamiltonian assumptions variable used in the current work for IBM-1 estimation pertain to the $^{102}_{42}Mo$

IBM-Parameter variable -MeV-	
ϵps (ε)	**0.4800**
a_0	**0.0100**
a_1	**0.0180**
a_2	**- 0.0160**
a_3	**0.0087**
a_4	**0.0073**

Table (1) indicates an incredibly precise alignment towards the Hamiltonian components Eq.(2) employed throughout this probe, leading to a high degree of concordance between the projected energy consumption in the planned construction and the real outcomes for $^{102}_{42}Mo$ [12,13], as can be seen in Figure (2).

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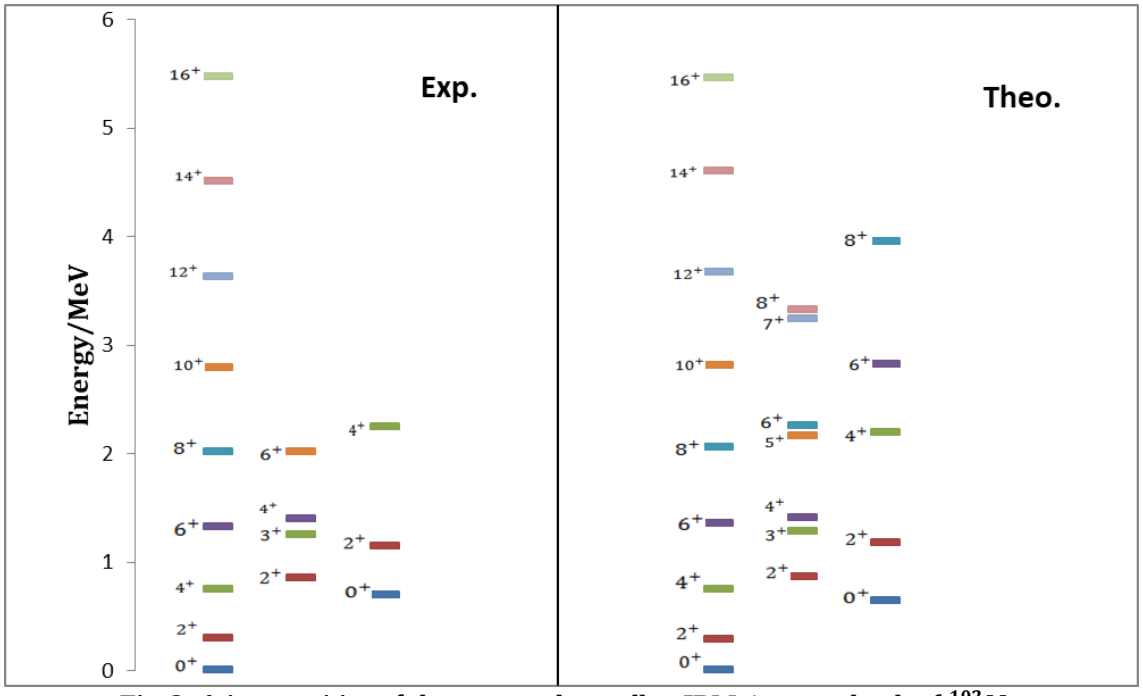

Fig. 2: A juxtaposition of the measured as well as IBM-1 energy levels of $^{102}_{42}Mo$

For the beneath the state of excitement of the studied $^{102}_{42}Mo$, the boson E2 operative in IBM-1, Its E2 values during transitions and quadrupole moments have been estimated using equation (3). According to the notion, the worth of the IBM-1's effective charge e_B was computed by normalizing to $B(E2: 2_1^+ \rightarrow 0_1^+)$ information gathered from trials. Table (2) shows the B(E2) values used in the current research utilizing the (IBMT-code).

Table 2: The juxtaposition of the estimated B(E2) transition (in unit e^2b^2) of $^{102}_{42}Mo$ with the measured amounts [14, 15], as well as quadrupole moments for the $Q2_1^+$ level provided in the final column.

Parameter	$\alpha_B = 0.12300$ e. b.	
Transitions	Experimental	Theoretical
$2_1^+ \rightarrow 0_1^+$	0.20948(2.55%)	0.2031
$2_1^+ \rightarrow 0_2^+$	0.1981(8.49%)	0.1898
$2_2^+ \rightarrow 0_1^+$		0.0052
$2_2^+ \rightarrow 0_2^+$		0.0661
$2_3^+ \rightarrow 0_1^+$		0.0001
$2_3^+ \rightarrow 0_2^+$		0.2243
$2_2^+ \rightarrow 2_1^+$		0.2581
$2_3^+ \rightarrow 2_1^+$		0.0022
$2_3^+ \rightarrow 2_2^+$		0.1176
$4_1^+ \rightarrow 2_1^+$	0.25195(5.10%)	0.2481
$4_1^+ \rightarrow 2_2^+$		0.0278
$4_1^+ \rightarrow 2_3^+$		0.1084
$4_2^+ \rightarrow 2_1^+$		0.0065
$4_2^+ \rightarrow 2_2^+$		0.2264
$4_2^+ \rightarrow 2_3^+$		0.0715
$4_3^+ \rightarrow 2_3^+$		0.3282
$4_2^+ \rightarrow 4_1^+$		0.2058
$6_1^+ \rightarrow 4_1^+$		0.4321
$8_1^+ \rightarrow 6_1^+$		0.4523
$Q2_1^+$		-0.6606

The current research's deductions for electric quadrupole transition probability estimates accord nicely with the measured values. The outstanding result was obtained from the ratio of excited states to the initial state, which was done by IBM-1 to $^{102}_{42}Mo$. It is clear that this extremely fits the experimental results as displayed in Figure (3). The present results show how the molybdenum isotope evidence is considered as gamma soft with O(6) chain group since it is unstable to gamma decay and the ratio of $\frac{E4_1^+}{E2_1^+}$ is close to (2.57) which is rough to experimental data[13]. The IBM-1 calculations of the abovementioned ratio values, overall, accord closely with the experimental energy ratio estimates.

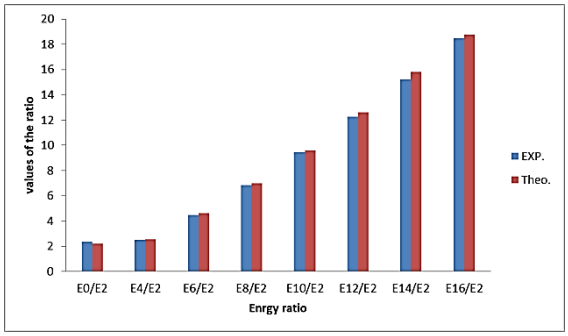

Fig. 3: Energy ratios are compared among theoretical and experimental results [12, 13] for $^{102}_{42}Mo$.

As previously stated, the general Hamiltonian may be considered in terms of the group U(6), which contains all unitary transformations in six dimensions. According to the current findings, the $^{102}_{42}Mo$ group chain is closed γ-soft deformed nuclei. The IBM-1 dynamical symmetry qualities may be explored by

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knowing the ratio $\frac{E4_1^+}{E2_1^+}$. If the proportion is close to (2.5), the nuclei will act like O(6) symmetry and going to be gamma softer. Figure (4) displays the molybdenum property of gamma unstable nuclei in the O(6) group chain.

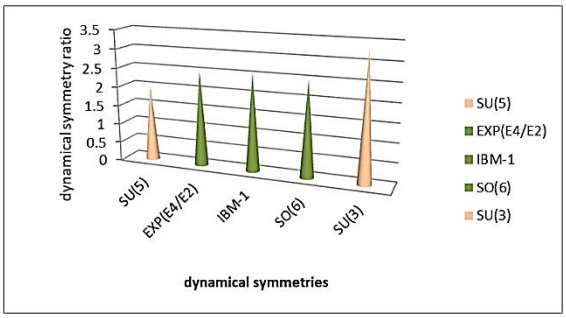

Fig. 4: comparison of experimental and theoretical values of the energy ratios ($\frac{E4_1^+}{E2_1^+}$) according to O(6) dynamical symmetry

The ratio $B(E2: j_i^+ \rightarrow j_f^+)/B(E2: j_i^+ \rightarrow j_f^+)$ exists, on the opposite conjunction, an aspect of the physical makeup of nuclei. Consequently, our current works ratio is with greatest conformity to data from experiments [14-16], as illustrated in Table (3):

Table 3: Correlation of actual and estimated B(E2) transition ratios for $^{102}_{42}Mo$.

Transitions ratio	Exp. [12, 13]	IBM-1
$\frac{4_1^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	1.21	1.22
$\frac{2_2^+ \rightarrow 2_1^+}{2_1^+ \rightarrow 0_1^+}$	--	1.27
$\frac{2_1^+ \rightarrow 0_2^+}{2_1^+ \rightarrow 0_1^+}$	0.94	0.9346

The good agreement comparison between theoretical and experimental B(E2) values encouraged us to try to extend the results to neutron-rich nuclei. The B(E2) transition ratios are useful tools for explaining nuclear structural features. These B(E2) ratios demonstrated that $^{102}_{42}$Mo is in the O(6) chain group's dynamical symmetry, with unstable to gamma. Figures (5) are compared among the observed [16] and IBM-1 ratios of the electric transition ratio for the molybdenum isotope.

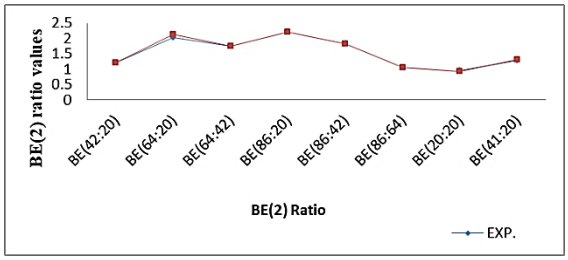

Fig. 5: the contrast of experimental and IBM-1 ratios of $B(E2: j_i^+ \rightarrow j_f^+)/B(E2: j_i^+ \rightarrow j_f^+)$ for $^{102}_{42}Mo$ nuclide

4. Conclusion

According to the current analysis, this isotope has nine bosons and gamma-soft nuclei with O(6) dynamical symmetry, with the $\frac{E4_1^+}{E2_1^+}$ ratio of (2.57) and $\frac{E6_1^+}{E2_1^+}$ ratio of (4.56) extremely near to experimental data of deformed nuclei in O(6) group chain. The results showed a high degree of consistency, demonstrating that the IBM-1 model can accurately describe the energy levels of this isotope. Furthermore, according to the current findings, additional investigation on $^{102}_{42}$Mo isotope B(E2) values will be required to identify the strength of E2 transitions.

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