Tikrit Journal of Pure Science (2023) 28 (5):154-165
Doi: https://doi.org/10.25130/tjps.v28i5.1589

TJPS

IASJ
المجلات الاكاديمية العراقية

Tikrit Journal of Pure Science
ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: http://tjps.tu.edu.iq/index.php/j

Independent Dominating Set on Chain of Fuzzy Graphs

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Keywords: A chain of fuzzy graphs, vertex identification, dominating set, independent set, independent dominating set.

ARTICLEINFO.

Article history:

-Received:	10 Jan. 2023
-Received in revised form:	13 Feb. 2023
-Accepted:	14 Feb. 2023
-Final Proofreading:	24 Oct. 2023
-Available online:	25 Oct. 2023

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ABSTRACT
In this paper, we applied some properties on chain fuzzy graphs, which comprise vertex identification. These properties are independent sets and independent dominant sets.

Tikrit Journal of Pure Science (2023) 28 (5):154-165
Doi: https://doi.org/10.25130/tjps.v28i5.1589

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المجموعات المهيمنة المستقلة على سلسلة البيانات المضببة

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الملخص

في هذا البحث، تم تعريف سلسلة البيانات المضببة التي تتكون من تطابق البيانات المضببة بواسطة الرؤوس. تم تطبيقها على بيانات الدارة ذات الخمس رؤوس. هذه البيانات تكون متشاكلة. وقمنا بتطبيق بعض الخواص عليها.

1. Introduction

The fuzzy graph is a mathematical application tool that allows users to easily express the connection between any two notions. The fuzzy graph idea and many fuzzy forms from graph theory topics were proposed by Rosenfeld, including paths, cycles, and connectedness, in 1975[1]. Mahmood and Ahmed first presented Schultz and Modified Schultz Polynomials for Vertex Identification Chain and Ring for Hexagon Graphs in 2021[2]. Somasundaram introduced the idea of dominance in fuzzy graphs in 1998[3]. In fuzzy graphs, Nagoorgani and Chandrasekaran studied fuzzy independent sets and fuzzy bipartite graphs in 2007. In 2013 Thakkar and Bosamiya addressed the essential idea of independence dominating [4]. we introduced in this work the notion of chain fuzzy graphs and we studied the relationship between independent and dominant. This paper contains fourth section, in section two, we introduced some basic concepts. Section three studied main results in this paper. In section four, we will present the results obtained from this research.

2.Basic Concepts:

A fuzzy graph denoted by $\mathcal{G} = (\mathcal{V}, \varrho, \varphi)$ on the clear graph $G = (V, E)$ is a nonempty set $\mathcal{V}$ and two function $\varrho: \mathcal{V} \to [0,1]$ and $\varphi: \mathcal{V} \times \mathcal{V} \to [0,1]$ such that $\forall\, u, v \in \mathcal{V}$, where $\varphi(u, v) \le \varrho(u) \wedge \varrho(v)$[5].

A path P in a fuzzy graph $\mathcal{G}$ is a collection of distinct vertices. $\mathcal{U}_0, \dots, \mathcal{U}_n$, where $\mathcal{U}_0 \neq \mathcal{U}_n$, and $n \geq 2$ such that $\varphi(\mathcal{U}_{i-1}, \mathcal{U}_i) > 0, i = 0, \dots, n$. We refer to the following pairs as the path's edges. The number of edges equals path length. A path P where $\mathcal{U}_0 = \mathcal{U}_n \; and \; n \geq 3$ is a cycle [6]. The weight of the weakest edge (the edge with least membership in path) is used to measure a path's strength. The strength of the connection between the vertices, u and v, is the maximum strength of all paths linking them, and it is denoted by $CCON_{\mathcal{G}}(u, v)$. If a path connects two vertices, they are said to be linked. A fuzzy graph $\mathcal{G} = (\mathcal{V}, \varrho, \varphi)$ is connected if $CCON_{\mathcal{G}}(u, v) > 0, \forall\, u, v \in \mathcal{V}$. An edge (u, v) is strong in $\mathcal{G}$ if $\varphi(u, v) > 0$, and $\varphi(u, v) \geq CCON_{\mathcal{G}-(u,v)}(u, v)$[1].

3. Main results:

3.1 The Vertex- Identification Chain Fuzzy Graphs:

The following is a formal definition of a chain of fuzzy graphs. **Definition3.1.1:** Assuming that $\{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n\}$, be a set of pairwise disjoint fuzzy graphs with vertices $u_i, v_i \in \mathcal{V}(\mathcal{G}_i)$ then the vertex-identification chain fuzzy graph , $C_{vf}(\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n \equiv C_{vf}(\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n: v_1 \cdot u_2; \dots; v_{n-1} \cdot u_n)$ of $\{\mathcal{G}_i\}_{i=1}^{n-1}$ with respect to the vertices $\{v_i, u_{i+1}\}_{i=1}^{n-1}$ is the graph obtained from the fuzzy graphs $\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_n$ by identifying the v_i vertex with the vertex u_{i+1} for all $i = 1,2 \dots n$. such that the vertex identification's weight value is $\varrho(w_i) = \max\{\varrho(v_i), \varrho(u_{i+1})\}$(denoted by $C_{vf}(C_n)$) (see Fig. 3.1):

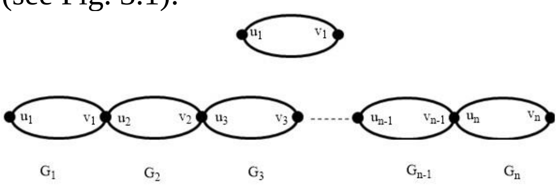


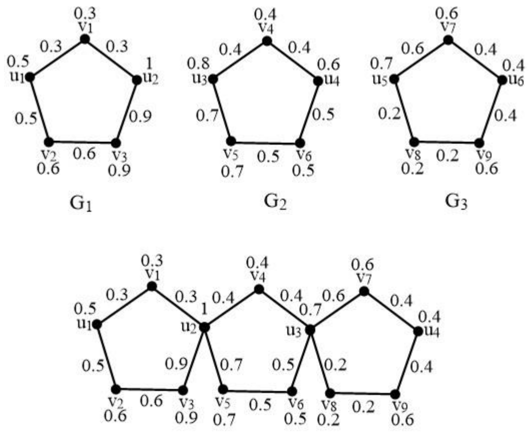
Fig. 3-1. Chain graphs

3.2. Dominating set of Chain Fuzzy Graphs:

Definition3.2.1[3]**:** A vertex $\mathcal{u}$ dominant other vertices in fuzzy graph $\mathcal{G} = (V, \varrho, \varphi)$. If $\varphi(\mathcal{u}, \mathcal{v}) = \varrho(\mathcal{u}) \wedge \varrho(\mathcal{v})$, for $\mathcal{u}, \mathcal{v} \in \mathcal{V}$.

A set $\mathfrak{D} \subseteq \mathcal{V}$ is said to be dominant set in $\mathcal{G}$ if for any $\mathcal{v} \in \mathcal{V} - \mathfrak{D}$, there exist $\mathcal{u} \in \mathfrak{D}$, such that, $\mathcal{u}$ dominates $\mathcal{v}$. The dominance number of $\mathcal{G}$ is the minimum cardinality of a set denoted by $\gamma(\mathcal{G})$.

Example3.2.1: In this example, all the edges are strong, which means there is no isolated vertices since isolated vertex does not dominate another vertex.



$C_{vf}(C_5)$

Fig.3-2

In this example, there are many dominating sets. One of these dominating sets is $\mathfrak{D} = \{u_1, u_2, u_3, u_4\}$ the vertices in $\mathfrak{D}$ that dominate the other vertices. $\mathfrak{D}$ is the set of vertex identification, and it is the minimum dominant set.

So, the dominance number is $\gamma\left(C_{vf}(C_n)\right) = 2.6$.

Theorem3.2.1: In $C_{vf}(C_n)$, if $n \leq 6$. Then $\gamma\left(C_{vf}(C_n)\right) = \sum_{i=1} \varrho(\mathcal{v}_i)$.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs with vertex identification $\mathcal{v}_i \in \mathfrak{D}$. Let $n \leq 6$. Since $n \leq 6$, so, any vertex in $\mathcal{V} - \mathfrak{D}$ adjacent to the vertex identifications, which means the vertex identifications $\mathcal{v}_i \in \mathfrak{D}$ dominate the vertices in $\mathcal{V} - \mathfrak{D}$. From the definition of domination number, we get the vertex identifications as the minimal dominating set of $C_{vf}(C_n)$. Then $\gamma\left(C_{vf}(C_n)\right) = \sum_{i=1} \varrho(\mathcal{v}_i)$.

In last example $\gamma\left(C_{vf}(C_n)\right) = 2.6$, which is the summation of the weight values of all the vertex identifications.

3.3 Independent set of Chain Fuzzy Graphs:

Definition3.3.1[7]**:** Suppose $\mathcal{G} = (\mathcal{V}, \varrho, \varphi)$ is a fuzzy graph. Any two varices in fuzzy graph are said to be fuzzy independent if there is no strong edge between them. If every vertex of a subset $\mathfrak{B}$ of $\mathcal{V}$ is fuzzy independent, then $\mathfrak{B}$ is said to be a fuzzy independent set for $\mathcal{G}$. Assumption $\mathcal{G} = (\mathcal{V}, \varrho, \varphi)$ is a fuzzy graph. When $\mathfrak{B}$ of $\mathcal{G}$ is the only fuzzy independent set with a cardinality higher than $\mathfrak{B}$, it is referred to as the maximal fuzzy independence set. $\mathcal{G}$'s fuzzy independence number has the highest number of nodes of all maximal fuzzy independent sets. (Denoted by $\beta(\mathcal{G})$).

Theorem3.3.1[7]: In a chain of fuzzy graphs, maximum independent set is a minimum dominant set.

Example3.3.1: In Fig 3-2, the set $\{u_1, u_2, u_3, u_4\}$, is a maximum independent set and a minimum dominant set.

Theorem3.3.2: In $C_{vf}(C_5)$, the vertex identification is an independence number.

Proof: Let $C_{vf}(C_5)$, be a chain of fuzzy graphs with vertex identification $\mathcal{v}_i$. Let $\mathfrak{B}$, be a maximal independent set. Since there is no strong edge between any vertex identification, the vertex identification is an fuzzy independent set.

Since $\mathfrak{B}$, is maximal fuzzy independent set, that's mean no other fuzzy independent set has a cardinality greater than S. S is the set of the vertex identification.

the vertex identification is an independence number.

3.4 Independent dominating set of Chain Fuzzy Graphs:

Definition3.4.1[4]**:** If the vertices of the dominant set $\mathfrak{D}$ of a fuzzy graph $\mathcal{G}$ are not connected to one another, then the set is an independent dominating set.

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If a fuzzy graph $\mathcal{G}$ has no suitable subset of its independent dominant set, then it is said to be a minimum independent dominating set.

A fuzzy graph's minimum independent dominating set cardinality is known as an independent domination number, (Provided by $\iota(\mathcal{G})$).

Theorem3.4.1 [8]**:** In a chain of fuzzy graphs, if $\mathfrak{D}$ is an independent dominant set, then $\mathfrak{D}$ is minimum dominant set and maximum independent set.

Remark 3.4.1: The converse the last theorem is true.

Example3.4.1: In Fig 3-2, $\mathfrak{D} = \{u_1, u_2, u_3, u_4\}$, is an independent dominant set, $\mathfrak{D}$ is a minimum dominant set and maximum independent set.

Theorem3.4.2: If $4 \leq n \leq 6$, then the vertex identification is an independent fuzzy dominating set.

Proof: let $4 \leq n \leq 6$. Let $\mathfrak{D}$, be the set of vertex identifications. Since every fuzzy graph in the chain is a cycle and $4 \leq n \leq 6$. Then the vertex identification is not adjacent. Every vertex identification in $\mathfrak{D}$ dominates a vertex in $\mathcal{V} - \mathfrak{D}$. $\mathfrak{D}$ is an independent fuzzy dominating set.

Theorem3.4.3: In a chain of fuzzy graphs, If $n \leq 6$.

Then $\gamma\left(C_{vf}(C_n)\right) = \beta\left(C_{vf}(C_n)\right)$.

Proof: Let $C_{vf}(C_n)$ be a chain of fuzzy graphs, with vertex identification $\mathcal{v}_i$. Let $\beta\left(C_{vf}(C_n)\right)$ be the maximal independent set, and $\gamma\left(C_{vf}(C_n)\right)$ be the minimum dominating set. If $n \leq 6$, every fuzzy graph in the chain has at least two vertices, but there are no strong edges between them. Since maximum independent set is a minimum dominant set.

From the description of domination number, and the definition of independence number, then $\gamma\left(C_{vf}(C_n)\right) = \beta\left(C_{vf}(C_n)\right)$.

Remark3.4.1: If $4 \leq n \leq 6$, then $\iota\left(C_{vf}(C_n)\right) = \sum_{i=1}^{n} \varrho(\mathcal{v}_i)$.

Theorem3.4.4: In a chain of fuzzy graphs, if $n \leq 6$ Then $\gamma\left(C_{vf}(C_n)\right) = \beta\left(C_{vf}(C_n)\right) = \iota\left(C_{vf}(C_n)\right)$

Proof: by theorem3.4.3, by theorem3.4.2 and last remark the theorem is true.

4. Conclusions

In this paper, we discussed the relationship between an independent set also a dominant set on a chain of fuzzy graphs. The vertex identification satisfies the condition of being independent and dominant, so it's an independent fuzzy dominant set.

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