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The β^m – Homeomorphism in Double Fuzzy Topological Spaces

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ABSTRACT

The purpose of this study is to introduce the concept of homeomorphism via β^m – closed set and study its behavior and properties in double fuzzy topological spaces. This objective is achieved through the definitions of df- β^m continuous functions and df- β^m closed functions. The results of this study represent important relationships and proofs, in addition to providing some necessary examples.

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التشاكل – $^m\beta$ في الفضاءات التبولوجية المضببة المزدوجة

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الملخص

في هذا البحث قدمنا مفهوم التشـــاكل التبولوجي .df- hom وكذلك مفهوم التشـــاكل التبولوجي من النمط .df- β^m hom من خلال تقديمنا لتعريف الدوال المغلقة والدوال المســتمرة على المجموعات المغلق في الفضــاءات التبولوجية المضــببة المزدوجة .(ř, š) –f β^m closed set وتم دراســة العلاقات بينهم من خلال مبرهنات والفرضيات والملاحظات والعديد من الامثلة الضرورية.

1. Introduction

Zadeh [1] presented his study on the concept of fuzzy set in 1965, which is a generalization of the classical sets. After that, in 1968 [2], Chang introduced his study on the fuzzy topological space. In 1993 [3], Atanassov introduced a new concept of sets, namely the intuitionistic fuzzy sets. On the basis of these sets, in 1997 [4], Coker introduced the fuzzy intuitionistic topological spaces. This study adopts the concept of Garcia and Rodabaugh [5] that the term double fuzzy topological spaces in mathematics is more appropriate than the intuitionistic fuzzy topological spaces. It studies df – β^m continuous functions, df – homeomorphism and df – β^m homeomorphism, as well as their properties and relationships.

2. Preliminaries

This section introduces some basic and essential concepts in this work. In this concern, a non-empty set is denoted by the symbol G and the closed unit interval $[\acute{0}, \acute{1}]$ by I, also $I_{ř}$ is denoted by $(\acute{0}, \acute{1}]$ and $I_{š}$ is denoted by $[\acute{0}, \acute{1})$. The family of all fuzzy sets is denoted by I^G. Hence, $\acute{0}$ and $\acute{1}$ represent the smallest and greatest fuzzy sets, respectively. For the fuzzy set $\tilde{\alpha} \in I^G$, $\acute{1} - \tilde{\alpha}$ denotes the complement of $\tilde{\alpha}$. The symbols $\leq$, $\wedge$ and $\vee$ represent the less or equal value, intersection and union, respectively. These symbols are used for fuzzy sets.

2.1. Definition: [1]

Let G be a non-empty set. A fuzzy set in G is characterized by its membership function $\delta_{\tilde{\alpha}}: G \to [\acute{0}, \acute{1}]$ where $\delta_{\tilde{\alpha}}(g)$ is interpreted as the degree of membership of element g in fuzzy set $\tilde{\alpha}$, for each $g \in G$. It is clear that $\tilde{\alpha}$ is completely determined by the set of tuples $\tilde{\alpha} = \{(g, \delta_{\tilde{\alpha}}(g)): g \in G\}$.

2.2. Definition: [5]

A double fuzzy topology $(\mathscr{J}, \mathscr{J}^*)$ on a non-empty set G is a pair of functions $\mathscr{J}, \mathscr{J}^*: I^G \to I$, which satisfies the following properties:

(i) $\mathscr{J}(\tilde{\alpha}) \leq \acute{1} - \mathscr{J}^*(\tilde{\alpha})$ for each $\tilde{\alpha} \in I^G$.

(ii) $\mathscr{J}(\tilde{\alpha}_1 \wedge \tilde{\alpha}_2) \geq \mathscr{J}(\tilde{\alpha}_1) \wedge \mathscr{J}(\tilde{\alpha}_2)$ and $\mathscr{J}^*(\tilde{\alpha}_1 \wedge \tilde{\alpha}_2) \leq \mathscr{J}^*(\tilde{\alpha}_1) \vee \mathscr{J}^*(\tilde{\alpha}_2)$ for each $\tilde{\alpha}_1, \tilde{\alpha}_2 \in I^G$.

(iii) $\mathscr{J}(\vee_{k \in r} \tilde{\alpha}_k) \geq \wedge_{k \in r} \mathscr{J}(\tilde{\alpha}_k)$ and $\mathscr{J}^*(\vee_{k \in r} \tilde{\alpha}_k) \leq \vee_{k \in r} \mathscr{J}^*(\tilde{\alpha}_k)$ for each $\tilde{\alpha}_k \in I^G$, $k \in r$.

The triplex $(G, \mathscr{J}, \mathscr{J}^*)$ is called double fuzzy topological spaces (dftss, for short).

2.3. Definition: [7]

Let $(G, \mathscr{J}, \mathscr{J}^*)$ be a dfts, then for each $ř \in I_{ř}$ and $š \in I_{š}$ and $\tilde{\alpha}, \delta \in I^G$, the double fuzzy closure $(C_{\mathscr{J},\mathscr{J}^*})$ and interior $(I_{\mathscr{J},\mathscr{J}^*})$ operator $C_{\mathscr{J},\mathscr{J}^*}, I_{\mathscr{J},\mathscr{J}^*}: I^G \times I_{ř} \times I_{š} \to I^G$ are defined as follows:

$$C_{\mathscr{J},\mathscr{J}^*}(\tilde{\alpha}, ř, š) = \wedge\{\delta \in I^G : \tilde{\alpha} \leq \delta, \mathscr{J}(\acute{1} - \delta) \geq ř, \mathscr{J}^*(\acute{1} - \delta) \leq š\}.$$

$$I_{\mathscr{J},\mathscr{J}^*}(\tilde{\alpha}, ř, š) = \vee\{\delta \in I^G: \delta \leq \tilde{\alpha}, \mathscr{J}(\delta) \geq ř, \mathscr{J}^*(\delta) \leq š\}.$$

2.4. Definition:

Let (G, $\mathscr{J}$, $\mathscr{J}^*$) be a dfts. For each α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$.

(1) α̃ is an (ř, š) –fuzzy open set ((ř, š) –f open) if $\mathscr{J}$(α̃) ≥ ř and $\mathscr{J}^*$(α̃) ≤ š and α̃ is an (ř, š) –fuzzy closed set ((ř, š) – f closed) if $\mathscr{J}$(1́ - α̃) ≥ ř and $\mathscr{J}^*$(1́ - α̃) ≤ š [9].

(2) α̃ is an (ř, š) –fuzzy semi open set ((ř, š) – fs open) if α̃ ≤ $C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) and an ((ř, š)-fuzzy semi closed set ((ř, š)-fs-closed), if $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) ≤ α̃ [9].

(3) α̃ is an (ř, š) –fuzzy preopen set ((ř, š) – fp open) if α̃ ≤ $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) and an (((ř, š) –fuzzy semi closed set ((ř, š)-fp closed), if $C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) ≤ α̃ [10].

(4) α̃ is an (ř, š) –fuzzy α open set ((ř, š) – f α open) if α̃ ≤ $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) and an (((ř, š) –fuzzy semi closed set ((ř, š) –fα closed), if $C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š), ř, š) ≤ α̃ [8].

(5) α̃ is an (ř, š) –fuzzy β open set ((ř, š) – fβ open) if α̃ ≤ $C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š), ř, š) and an (((ř, š) – fuzzy β closed set ((ř, š) – fβ closed), if $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$($I_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š), ř, š) ≤ α̃ [12].

(6) α̃ is called (ř, š) – generalized fuzzy closed ((ř, š) – gf closed) set if $C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š) ≤ δ, whenever, α̃ ≤ δ and $\mathscr{J}$(δ) ≥ ř, $\mathscr{J}^*$(δ) ≤ š. Complement of (ř, š) – gf closed set is an (ř, š) – gf open set [8].

(7) α̃ is called (ř, š) – fuzzy $α^m$ closed ((ř, š) –f $α^m$ closed) set if $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) ≤ δ, whenever, α̃ ≤ δ and δ is an (ř, š)–fα open. α̃ is called an (ř, š) – fuzzy $α^m$ open ((ř, š) – f $α^m$ open) if 1́ – α̃ is an (ř, š) – f $α^m$ closed set [11].

2.5. Definition: [12]

Let (G, $\mathscr{J}$, $\mathscr{J}^*$) be a dfts, for each α̃, δ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$. α̃ is called an (ř, š) –fuzzy $β^m$ closed ((ř, š) – f$β^m$ closed) set if $I_{\mathscr{J},\mathscr{J}^*}$($C_{\mathscr{J},\mathscr{J}^*}$(α̃, ř, š), ř, š) ≤ δ, whenever α̃ ≤ δ and δ is an (ř, š) – fβ open. α̃ is called an (ř, š) – fuzzy $β^m$ open ((ř, š) – f$β^m$ open) if 1́ – α̃ is an (ř, š) – f$β^m$ closed set.

2.6. Definition

Let ƒ: (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) → (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$), then ƒ is said to be:

(1) double fuzzy – closed (df – closed) function if image for every (ř, š) – f closed set is an (ř, š) – f closed set in H whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$ [9].
(2) double fuzzy – continuous (df – con) function if inverse image for every (ř, š) – f closed set is an (ř, š) – f closed set in G whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$ [9].
(3) double fuzzy – semi continuous (df – s con) function if inverse image for every (ř, š) – f closed set is an (ř, š) – f s closed set in G whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$ [6].
(4) double fuzzy – generalized continuous (df – g con) function if inverse image for every (ř, š) – f closed set is an (ř, š) –g f closed set in G whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$ [6].
(5) double fuzzy – $α^m$ continuous (df – $α^m$ con) function if inverse image for every (ř, š) – f closed set is an (ř, š) – f $α^m$ closed set in G whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$ [11].

In this part, the double fuzzy $β^m$ – homeomorphism is introduced.

3.1. Definition

Let ƒ: (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) → (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$), then ƒ is said to be double fuzzy – $β^m$ closed (df – $β^m$ closed) function if image for every (ř, š) – f closed set is an (ř, š) – f $β^m$ closed set in H whenever α̃ ∈ I^G, ř ∈ $I_ř$ and š ∈ $I_š$.

3.2. Example

Let G={c, d}, H= {k, v} and ($\mathscr{J}_1$(φ), $\mathscr{J}_1^*$(φ)) is defined on G by:

$$\mathcal{J}_1(\varphi)=\begin{cases} \acute{1}, & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathcal{J}_1^*(\varphi)=\begin{cases} \acute{0} & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \acute{1} & \text{otherwise.} \end{cases}$$

Such that,

$$\varphi_1(c) = 0.4, \quad \varphi_1(d) = 0.6,$$

Also, ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$) is defined on H by:

$$\mathcal{J}_2(\Upsilon)=\begin{cases} \acute{1}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h)=\Upsilon_1 \\ \frac{1}{4}, & \text{if } \Upsilon(h)=\Upsilon_2 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathcal{J}_2^*(\Upsilon)=\begin{cases} \acute{0}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h)=\Upsilon_1 \\ \frac{3}{4}, & \text{if } \Upsilon(h)=\Upsilon_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that,

$$\Upsilon_1(k) = 0.6, \quad \Upsilon_1(v) = 0.3.$$

And,

$$\Upsilon_2(k) = 0.4, \quad \Upsilon_2(v) = 0.7.$$

When, the function ƒ between two dfts ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$) is defined by: ƒ: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) → (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$), as ƒ(c) = k and ƒ(d) = v.

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(\acute{1} - \varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\acute{1} - \varphi_1) \leq \frac{1}{2}$, such that ƒ$(\acute{1} - \varphi_1)$ = ƒ$(\varphi_1^c) = (c_{0.6}, d_{0.4}) \leq \Upsilon_1$, where Υ_1 is an $(\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, ƒ(φ_1^c) is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, $\mathcal{F}$ is df - β^m closed function.

3.3. Remark

Every df – closed function is df – β^m closed function, but need not conversely.

3.4. Example

Refer to example 3.2., a fuzzy set φ_1 is taken for dfts ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and the fuzzy sets Υ_1 and Υ_2 for dfts ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$), such that

$$\varphi_1(c) = 0.5, \quad \varphi_1(d) = 0.6$$

And,

$$\Upsilon_1(k) = 0.6, \quad \Upsilon_1(v) = 0.3$$

$$\Upsilon_2(k) = 0.4, \quad \Upsilon_2(v) = 0.6.$$

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(\acute{1} - \varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\acute{1} - \varphi_1)$. Now, ƒ$(\acute{1} - \varphi_1)$ = ƒ$(\varphi_1^c) = (c_{0.5}, d_{0.4}) \leq \Upsilon_1$, where Υ_1 is an $(\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, ƒ(φ_1^c) is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, ƒ is df - β^m closed function. But ƒ is not df - closed function, since ƒ(φ_1^c) is not an $(\frac{1}{2}, \frac{1}{2})$ – f closed set.

3.5. Definition

Let ƒ: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) → (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) be a function between these dftss (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) and (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$). Then, ƒ is said to be a double fuzzy – β^m continuous (df– β^m con) function if inverse image for every $\mathcal{J}_2(\acute{1} - \tilde{\alpha}) \geq \check{r}$, and $\mathcal{J}_2^*(\acute{1} - \tilde{\alpha}) \leq \check{s}$ is an $(\check{r}, \check{s})$ – fβ^m closed set in M, whenever $\tilde{\alpha} \in I^G$, $\check{r} \in I_{\check{r}}$ and $\check{s} \in I_{\check{s}}$.

3.6. Example

Let G={c, d}, H= {k, v} and ($\mathscr{J}_1(\varphi)$, $\mathscr{J}_1^*(\varphi)$) is defined on G by:

$$\mathscr{J}_1(\varphi)=\begin{cases} \acute{1}, & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g)=\varphi_2 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathscr{J}_1^*(\varphi)=\begin{cases} \acute{0}, & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g)=\varphi_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that, $\varphi_1(c) = 0.5$, $\varphi_1(d) = 0.4$,

And, $\varphi_2(c) = 0.4$, $\varphi_2(d) = 0.7$.

Also, ($\mathscr{J}_2(\Upsilon)$, $\mathscr{J}_2^*(\Upsilon)$) is defined on H by:

$$\mathscr{J}_2(\Upsilon)=\begin{cases} \acute{1}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h)=\Upsilon_1 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathscr{J}_2^*(\Upsilon)=\begin{cases} \acute{0} & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h)=\Upsilon_1 \\ \acute{1} & \text{otherwise.} \end{cases}$$

Such that, $\Upsilon_1(k) = 0.6$ and $\Upsilon_1(v) = 0.4$,

When the function ƒ between the two dftss ($\mathscr{J}_1(\varphi)$, $\mathscr{J}_1^*(\varphi)$) and ($\mathscr{J}_2(\Upsilon)$, $\mathscr{J}_2^*(\Upsilon)$) is defined by: ƒ: (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) → (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$), as ƒ(c) = k and ƒ(d) = v. So, $\mathscr{J}_2(\Upsilon_1) \geq \frac{1}{2}$, $\mathscr{J}_2^*(\Upsilon_1) \leq \frac{1}{2}$, $ƒ^{-1}(\Upsilon_1^c) = (k_{0.4}, v_{0.6}) \leq \varphi_1$. Then, $ƒ^{-1}(\Upsilon_1^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. This implies that ƒ is df – β^m con function.

3.7. Definition

Let (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) and (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$) be dftss. A bijection function ƒ: (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) → (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$) is said to be a double fuzzy – homeomorphism (df – hom) if ƒ and $ƒ^{-1}$ are df– con functions.

3.8. Example

Let G ={c, d}, H = {k, v} and ($\mathscr{J}_1(\varphi)$, $\mathscr{J}_1^*(\varphi)$) is defined on G by:

$$\mathscr{J}_1(\varphi)=\begin{cases} \acute{1}, & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathscr{J}_1^*(\varphi)=\begin{cases} \acute{0} & \text{if } \varphi \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g)=\varphi_1 \\ \acute{1} & \text{otherwise.} \end{cases}$$

Such that,

$\varphi_1(c) = 0.4$, $\varphi_1(d) = 0.6$,

Also, ($\mathscr{J}_2(\Upsilon)$, $\mathscr{J}_2^*(\Upsilon)$) is defined on H by:

$$\mathfrak{I}_2(\Upsilon)=\begin{cases} \acute{1}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(v)=\Upsilon_1 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathfrak{I}_2^*(\Upsilon)=\begin{cases} \acute{0} & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(v)=\Upsilon_1 \\ \acute{1} & \text{otherwise.} \end{cases}$$

Such that,

$\Upsilon_1(k) = 0.4$, $\Upsilon_1(v) = 0.6$, When the bijection function ƒ between the two dftss ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$) is defined by: ƒ: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) → (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$), as f(c) = k and f(d) = v.

So, $\mathcal{J}_2(\Upsilon_1) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\Upsilon_1) \leq \frac{1}{2}$, so $\mathcal{J}_2(\acute{1} - \Upsilon_1) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\acute{1} - \Upsilon_1) \leq \frac{1}{2}$. Then, ƒ is df - con function. And $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, so $\mathcal{J}_1(\acute{1} - \varphi_1) \geq \frac{1}{2}$, $\mathfrak{J}_1^*(\acute{1} - \varphi_1) \leq \frac{1}{2}$.
Then, $ƒ^{-1}$ is df - con function. That is ƒ is df – hom.

3.9. Remark

Every df – hom is df – closed (df – β^m closed) function, but need not conversely.

3.10. Example

Refer to example 3.2., a fuzzy set φ_1 is taken for dfts ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and the fuzzy sets Υ_1 and Υ_2 for dfts ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$), such that:

$$\varphi_1(c) = 0.5, \quad \varphi_1(d) = 0.5$$

And,

$$\Upsilon_1(k) = 0.5, \quad \Upsilon_1(v) = 0.5$$

$$\Upsilon_2(k) = 0.4, \quad \Upsilon_2(v) = 0.6.$$

So, $\mathcal{J}_1(\varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\varphi_1) \leq \frac{1}{2}$, then $\mathcal{J}_1(\acute{1} - \varphi_1) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\acute{1} - \varphi_1)$.Then, ƒ is df – closed function, and since every df – closed function is df – β^m closed function, so ƒ is df – β^m closed function. But, ƒ is not df – hom, since ƒ is not df – con function.

3.11. Remark

Every df – hom is df – con (df – β^m con, df – α^m con, df – semi con, df – g con) function, but need not conversely.

3.12. Example

Refer to example 3.6., a fuzzy set φ_1 is taken for dfts ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and the fuzzy sets Υ_1 and Υ_2 for dfts ($\mathcal{J}_2(\Upsilon)$, $\mathcal{J}_2^*(\Upsilon)$), such that: $\varphi_1(c) = 0.5$, $\varphi_1(d) = 0.5$,

$$\varphi_2(c) = 0.4, \quad \varphi_2(d) = 0.7.$$

And $\Upsilon_1(k) = 0.5$, $\Upsilon_1(v) = 0.5$

ƒ is df – con (df – β^m con, df – α^m con, df – semi con, df – g con) function, but ƒ is not df – hom, since $ƒ^{-1}$ is not df – con function.

3.13. Definition

A bijection function ƒ: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) → (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) is said to be a double fuzzy – β^m homeomorphism (df – β^m hom) if ƒ and $ƒ^{-1}$ are df– β^m con functions.

3.14. Example

Let G={c, d}, H= {k, v} and ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) is defined on G by:

$$\mathscr{J}_1(\varphi) = \begin{cases} \acute{1}, & \text{if } \varphi \in \{\acute{0}, \acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g) = \lambda_2 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathscr{J}_1^*(\varphi) = \begin{cases} \acute{0}, & \text{if } \varphi \in \{\acute{0}, \acute{1}\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g) = \varphi_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that, $\varphi_1(c) = 0.5$, $\varphi_1(d) = 0.4$,

And, $\varphi_2(c) = 0.4$, $\varphi_2(d) = 0.7$.

Also, $(\mathscr{J}_2(\Upsilon), \mathscr{J}_2^*(\Upsilon))$ is defined on H by:

$$\mathscr{J}_2(\Upsilon) = \begin{cases} \acute{1}, & \text{if } \Upsilon \in \{\acute{0}, \acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h) = \Upsilon_1 \\ \frac{1}{4}, & \text{if } \Upsilon(h) = \Upsilon_2 \\ \acute{0}, & \text{otherwise.} \end{cases} \quad , \quad \mathscr{J}_2^*(\Upsilon) = \begin{cases} \acute{0}, & \text{if } \Upsilon \in \{\acute{0}, \acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(gh) = \Upsilon_1 \\ \frac{3}{4}, & \text{if } \Upsilon(h) = \Upsilon\varphi_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that, $\Upsilon_1(k) = 0.6$, $\Upsilon_1(v) = 0.4$,

And, $\Upsilon_2(k) = 0.5$, $\Upsilon_2(v) = 0.6$.

Now, ƫ is df- β^m con function, since $ƫ^{-1}(\acute{1} - \Upsilon_1)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. And, $ƫ^{-1}$ is df- β^m con function, since $(ƫ^{-1})^{-1}(\acute{1} - \varphi_1)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. That is, ƫ is df– β^m hom.

3.15. Theorem

Every df – hom is df – β^m hom, but need not conversely.

Proof

Let a function ƫ: $(G, \mathscr{J}_1, \mathscr{J}_1^*) \to (H, \mathscr{J}_2, \mathscr{J}_2^*)$ be a df – hom.

Then, ƫ and $ƫ^{-1}$ are df– con function. This implies that ƫ and $ƫ^{-1}$ are df– β^m con function.
That is a function ƫ is a df – β^m hom.

The following example demonstrates that the converse of this theorem is not true.

3.16. Example

Refer to example 3.14., ƫ is df– β^m hom. But, ƫ is not df– hom, since $ƫ^{-1}(\acute{1} - \Upsilon_1)$ is not $(\frac{1}{2}, \frac{1}{2})$ – f closed set.

3.17. Theorem

Let ƫ: $(G, \mathscr{J}_1, \mathscr{J}_1^*) \to (H, \mathscr{J}_2, \mathscr{J}_2^*)$ be a function, if ƫ is df – bijective function, then the following statements are equivalent:

i) $ƫ^{-1}$: $(H, \mathscr{J}_2, \mathscr{J}_2^*) \to (G, \mathscr{J}_1, \mathscr{J}_1^*)$ is df – β^m con function.
ii) ƫ is df – β^m open function.
iii) ƫ is df – β^m closed function.

Proof

i → ii: Let $\mathcal{J}_1$(α̃) ≥ ŕ and $\mathcal{J}_1^*$(α̃) ≤ ś, then $\mathcal{J}_1$(1́ - α̃) ≥ ř and $\mathcal{J}_1^*$(1́ - α̃) ≤ š, where ř ∈ Iř and š ∈ Iš. Since, $f̧^{-1}$ is df – β^m con function. Then, $(f̧^{-1})^{-1}$(1́ - α̃)) is an (ř, š) – f β^m closed set in H.
This implies that f̧(1́ - α̃) is an (ř, š) – f β^m closed set in H. That is 1́ - f̧(α̃) is an (ř, š) – fβ^m closed set in H.
So, f̧(α̃) is an (ř, š) – f β^m open set in H. This implies that f̧ is df – β^m open function.

ii → iii: Let $\mathcal{J}_1$(1́ - δ) ≥ ř and $\mathcal{J}_1^*$(1́ - δ) ≤ š, then $\mathcal{J}_1$(1́ - (1́ - δ)) ≥ ř and $\mathcal{J}_1^*$(1́ - (1́ - δ)) ≤ š, where ř ∈ $I_ř$ and š ∈ $I_š$. Since f̧ is df – β^m open function, then f̧(1́ - (1́ - δ)) is an (ř, š) – f β^m open set in H. This implies that 1́ - f̧(1́ - δ) is an (ř, š) - f β^m open set in H. That is f̧(1́ - δ) is an (ř, š) – f β^m closed set in H.
This implies that f̧ is df – β^m closed function.

iii → i: Let $\mathcal{J}_1$(1́ - γ) ≥ ř and $\mathcal{J}_1^*$(1́ - γ) ≤ š, where ř ∈ $I_ř$ and š ∈ $I_š$.
Since f̧ is df – β^m closed function, then f̧(1́ - γ)) is an (ř, š) – f β^m closed set in H. That is $(f̧^{-1})^{-1}$(1́ - γ)) is an (ř, š) – f β^m closed set in H. That is $f̧^{-1}$ is df – β^m con function.

3.18. Theorem

Let a function f̧: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) → (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) be df – β^m hom. Then, a function f̧ is df – hom if G and H represent df – $(\mathcal{J}, \mathcal{J}^*)_{T\beta m}$ space.

Proof

Let $\mathcal{J}_2$(1́ - ᾱ) ≥ ř and $\mathcal{J}_2^*$(1́ - ᾱ) ≤ š, where ř ∈ $I_ř$ and š ∈ $I_š$. Since, f̧ is df – β^m hom, then f̧ is df – β^m con function. That is, $f̧^{-1}$(1́ - ᾱ) is an (ř, š) – f β^m closed set in G. Now, since G is df – $(\mathcal{J}, \mathcal{J}^*)_{T\beta m}$ space, this implies that $f̧^{-1}$(1́ - ᾱ) is an (ř, š) – f closed set in G.
Hence, f̧ is df –con function, by hypothesis, $f̧^{-1}$: (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) → (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) is df – β^m con function.
Let $\mathcal{J}_1$(1́ - γ) ≥ ř and $\mathcal{J}_1^*$(1́ - γ) ≤ š, where ř ∈ $I_ř$ and š ∈ $I_š$.
Then, $(f̧^{-1})^{-1}$(1́ - γ)) = f̧(1́ - γ) is an (ŕ, ś) – fβ^m closed set in H, since H is df – $(\mathcal{J}, \mathcal{J}^*)_{T\beta m}$ space. That is f̧(1́ - γ) is an (ř, š) – f closed set in H. This implies that $f̧^{-1}$ is df – con function. Therefore, a function f̧ is df – hom.

3.19. Proposition

The composition of two df – β^m hom does not need df – β^m hom, as clarified in the following example.

3.20. Example

Let G={c, d}, H = {k, h} and F = {q, w}, ($\Im_1$(φ), $\Im_1^*$(φ)) is defined on M by:

$$\mathcal{J}_1(\varphi)=\begin{cases} 1́, & \text{if } \varphi \in \{0́, 1́\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{1}{4}, & \text{if } \varphi(g) = \varphi_2 \\ 0́, & \text{otherwise.} \end{cases} \quad , \quad \mathcal{J}_1^*(\varphi)=\begin{cases} 0́, & \text{if } \varphi \in \{0́, 1́\}, \\ \frac{1}{2}, & \text{if } \varphi(g) = \varphi_1 \\ \frac{3}{4}, & \text{if } \varphi(g) = \varphi_2 \\ 1́, & \text{otherwis.} \end{cases}$$

Such that,

$$\varphi_1(c) = 0.2, \quad \varphi_1(d) = 0.4,$$

And,
$$\varphi_2(c) = 0.8, \quad \varphi_2(d) = 0.6.$$

Also, ($\mathcal{J}_2$(ϒ), $\mathcal{J}_2^*$(ϒ)) is defined on H by:

$$\mathcal{J}_2(\Upsilon)=\begin{cases} \acute{1}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h) = \Upsilon_1 \\ \frac{1}{4}, & \text{if } \Upsilon(h) = \Upsilon_2 \\ \acute{0}, & \text{otherwise.} \end{cases}, \quad \mathcal{J}_2^*(\Upsilon)=\begin{cases} \acute{0}, & \text{if } \Upsilon \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \Upsilon(h) = \Upsilon_1 \\ \frac{3}{4}, & \text{if } \Upsilon(h) = \Upsilon_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that,

$$\Upsilon_1(k) = 0.4, \quad \Upsilon_1(v) = 0.9.$$

And, $$\Upsilon_2(k) = 0.4, \quad \Upsilon_2(v) = 0.3.$$

Also, ($\mathcal{J}_3(\vartheta)$, $\mathcal{J}_3^*(\vartheta)$) is defined on F by:

$$\mathfrak{I}_3(\vartheta)=\begin{cases} \acute{1}, & \text{if } \vartheta \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \vartheta(f) = \vartheta_1 \\ \frac{1}{4}, & \text{if } \vartheta(f) = \vartheta_2 \\ \acute{0}, & \text{otherwise.} \end{cases}, \quad \mathfrak{I}_3^*(\vartheta)=\begin{cases} \acute{0}, & \text{if } \vartheta \in \{\acute{0},\acute{1}\}, \\ \frac{1}{2}, & \text{if } \vartheta(f) = \vartheta_1 \\ \frac{3}{4}, & \text{if } \vartheta(f) = \vartheta_2 \\ \acute{1}, & \text{otherwis.} \end{cases}$$

Such that,

$$\vartheta_1(q) = 0.6, \quad \vartheta_1(w) = 0.6.$$

And, $$\vartheta_2(q) = 0.4, \quad \vartheta_2(w) = 0.8.$$

When the bijection function $\mathcal{F}$ between the two dftss ($\mathfrak{I}_1(\varphi)$, $\mathfrak{I}_1^*(\varphi)$) and ($\mathfrak{I}_2(\Upsilon)$, $\mathfrak{I}_2^*(\Upsilon)$) is defined by:

$\mathfrak{f}$: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) ⟶ (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) as, $\mathfrak{f}$(c) = k and $\mathfrak{f}$(d) = v.

Now, $\mathcal{J}_2(\Upsilon_1) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\Upsilon_1) \leq \frac{1}{2}$, so $\mathcal{J}_2(\acute{1} - \Upsilon_1) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\acute{1} - \Upsilon_1) \leq \frac{1}{2}$,

$\mathfrak{f}^{-1}(\acute{1} - \Upsilon_2) \leq \varphi_2$, φ_2 is an $(\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, $\mathfrak{f}^{-1}(\Upsilon_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, $\mathfrak{f}$ is df - β^m closed function. And $\mathfrak{f}^{-1}$ is df - β^m con function, since $\mathfrak{I}_1(\varphi_2) \geq \frac{1}{2}$, $\mathfrak{I}_1^*(\varphi_2) \leq \frac{1}{2}$, so $\mathfrak{I}_1(\acute{1} - \varphi_2) \geq \frac{1}{2}$, $\mathfrak{I}_1^*(\acute{1} - \varphi_2) \leq \frac{1}{2}$, $(\mathfrak{f}^{-1})^{-1}(\acute{1} - \varphi_2) \leq \Upsilon_2$, Υ_2 is an $\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, $(\mathfrak{f}^{-1})^{-1}(\varphi_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, $\mathfrak{f}^{-1}$ is df - β^m con function. That is $\mathfrak{f}$ is df - β^m hom. And, the bijection function g between the two dftss ($\mathfrak{I}_2(\Upsilon)$, $\mathfrak{I}_2^*(\Upsilon)$) and ($\mathfrak{I}_3(\vartheta)$, $\mathfrak{I}_3^*(\vartheta)$) is defined by: g: (H, $\mathcal{J}_2$, $\mathcal{J}_2^*$) ⟶ (F, $\mathcal{J}_3$, $\mathcal{J}_3^*$) as, g(k) = q and g(v) = w. $\mathcal{J}_3(\vartheta_1) \geq \frac{1}{2}$, $\mathcal{J}_3^*(\vartheta_1) \leq \frac{1}{2}$, so $\mathcal{J}_3(\acute{1} - \vartheta_1) \geq \frac{1}{2}$, $\mathcal{J}_3^*(\acute{1} - \vartheta_1) \leq \frac{1}{2}$, $g^{-1}(\acute{1} - \vartheta_1) \leq \Upsilon_2$, Υ_2 is an $\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, $g^{-1}(\vartheta_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, g is df - β^m closed function. And g is df - β^m con function, since $\mathcal{J}_2(\Upsilon_2) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\Upsilon_2) \leq \frac{1}{2}$, so $\mathcal{J}_2(\acute{1} - \Upsilon_2) \geq \frac{1}{2}$, $\mathcal{J}_2^*(\acute{1} - \Upsilon_2) \leq \frac{1}{2}$, $(g^{-1})^{-1}(\acute{1} - \Upsilon_2) \leq \vartheta_1$, ϑ_1 is an $(\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, $(g^{-1})^{-1}(\Upsilon_2^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, g^{-1} is df - β^m con function. That is g^{-1} is df - β^m hom.

Now, the bijection function $\mathfrak{f}$ between the two dftss ($\mathcal{J}_1(\varphi)$, $\mathcal{J}_1^*(\varphi)$) and ($\mathcal{J}_3(\vartheta)$, $\mathcal{J}_3^*(\vartheta)$) is defined by: g o $\mathfrak{f}$: (G, $\mathcal{J}_1$, $\mathcal{J}_1^*$) ⟶ (F, $\mathcal{J}_3$, $\mathcal{J}_3^*$) as, g o $\mathfrak{f}$(c) = q and g o $\mathfrak{f}$(d) = f. $\mathcal{J}_3(\vartheta_1) \geq \frac{1}{2}$, $\mathcal{J}_3^*(\vartheta_1) \leq \frac{1}{2}$, so $\mathcal{J}_3(\acute{1} - \vartheta_1) \geq \frac{1}{2}$, $\mathcal{J}_3^*(\acute{1} - \vartheta_1) \leq \frac{1}{2}$,

$(g \circ \mathfrak{f})^{-1}(\acute{1} - \vartheta_1) \leq \acute{1}$, $\acute{1}$ is an $\frac{1}{2}, \frac{1}{2})$ – fβ open set. So, $(g \circ \mathfrak{f})^{-1}(\vartheta_1^c)$ is an $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, (g o $\mathfrak{f}$) is df - β^m con function. But $(g \circ \mathfrak{f})^{-1}$ is not df - β^m con function. Since $\mathcal{J}_1(\varphi_2) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\varphi_2) \leq \frac{1}{2}$, so $\mathcal{J}_1(\acute{1} - \varphi_2) \geq \frac{1}{2}$, $\mathcal{J}_1^*(\acute{1} - \varphi_2) \leq \frac{1}{2}$,
$((g \circ \mathfrak{f})^{-1})^{-1}(\acute{1} - \varphi_2) \leq \varphi_1$, φ_1 is an $(\frac{1}{2}, \frac{1}{2})$ – fβ open set. So,
$(g \circ \mathfrak{f})^{-1}(\varphi_2^c)$ is not $(\frac{1}{2}, \frac{1}{2})$ – f β^m closed set. Hence, (g o $\mathfrak{f}$) is not df - β^m hom.

3.21. Proposition

The composition of two df – β^m hom needs df – β^m hom if H is df – $(\mathscr{J}, \mathscr{J}^*)_{T\beta m}$ space.

Proof

Let a function ƒ: (G, $\mathscr{J}_1$, $\mathscr{J}_1^*$) → (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$) and g: (H, $\mathscr{J}_2$, $\mathscr{J}_2^*$) → (F, $\mathscr{J}_3$, $\mathscr{J}_3^*$) be df – β^m hom.
And let $\mathscr{J}_3$(1́ - α̃) ≥ ř and $\mathscr{J}_3^*$(1́ - α̃) ≤ š, since g is df- df – β^m hom.
So, g^{-1}(1́ - α̃) is an (ř, š) – f β^m closed set in H. Since a space H is df – $(\mathscr{J}, \mathscr{J}^*)_{T\beta m}$ space.
Then, g^{-1}(1́ - α̃) is an (ř, š) – f closed set in H, since ƒ is df- β^m hom.
This implies that $ƒ^{-1}(g^{-1}$(1́ - α̃)) is an (ř, š) –f β^m closed set in G.
Then, $ƒ^{-1}(g^{-1}$(1́ - α̃)) = $(g \circ ƒ)^{-1}$(1́ - α̃) is an (ř, š) –f β^m closed set in G. Therefore, g o ƒ is df– β^m hom.

3. Conclusion

In this study, the ideas and definitions of symmetry on functions in the fuzzy double topological space are presented and studied. This definition is generalized to a definition of a new concept, which is df–β^m homemorphism. In addition, the study develops the relationship between these new types of symmetry of functions in double fuzzy topological spaces with some other types of functions that were previously studied through theorems and a number of examples. Finally, the results are as follows:

(1) Every df – hom is df – con (df – β^m con, df – α^m con, df – semi con, df – g con) function. (2)
Every df – hom is df – β^m hom.

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