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The Stability for Limit Cycle of Lomax Autoregressive Model

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ABSTRACT

In this research, we proposed a new non-linear model known as the Lomax autoregressive model, which is based on the cumulative distribution function of the Lomax distribution. Using the local linearization approximation technique, we found stability for the proposed first-order model's limit cycle. Then, we generalized the conditions for the Lomax autoregressive model of order p and found stability for the limit cycle of period (q > 1). Some examples illustrate the state of stability, and we plot the trajectory for the model of different initial values.

Keywords: Limit cycle, Lomax autoregressive model, Local linearization method, Stability, non-linear time series.

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شروط استقرارية دورة النهاية لأنموذج لوماكس للانحدار الذاتي

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الملخص

في هذا البحث اقترحنا أنموذج لا خطي جديد يعرف بأنموذج لوماكس للانحدار الذاتي الذي يعتمد على دالة التوزيع التراكمي لتوزيع لوماكس، وقد وجدنا الاستقرارية لدورة النهاية للأنموذج المقترح من الرتبة الأولى باستخدام تقنية التقريب بالخطية المحلية، ثم قمنا بتعميم الشروط لأنموذج لوماكس من الرتبة p والعثور على حالة الاستقرارية لدورة النهاية للفترة (q > 1) . وتم اعطاء بعض الامثلة التي توضح حالة الاستقرارية. وتم رسم مسارات الحل للأنموذج لقيم ابتدائية مختلفة.

1. INTRODUCTION

One of the nonlinear characteristics of a system or model with a periodic solution is a limit cycle. A helpful tool for analysing the behaviour of paths close to each point in a limit cycle is the local linearization approach. In a continuous-time dynamical system, a limit cycle is a closed curve that symbolizes the system's periodic solution. When the orbit of the solution approaches the closed curve as (t→∞), the limit cycle is stable. On the other hand, in a discrete-time dynamical system or model, a limit cycle is a finite collection of points $\{x_t, x_{t-1}, \dots x_{t-q}\}$ such that $(x_t = x_{t-q})$ where q is the smallest positive integer bigger than one. It has been studied by some researchers about the limit cycle steady state for nonlinear time series models. In (1977) examined the exponential autoregressive model[1]. In 1982 Time series models were utilized to study the statistical analysis of disturbed limit cycle processes[2]. In (2010) has been studied the Cauchy autoregressive model[3]. In (2012) has been studied the stability of nonlinear autoregressive models with trigonometric functions[4]. A study of stability conditions for the autoregressive model (Burr X) was proposed in (2019)[5]. In (2020) has been studied the dynamical approach to studying the stability condition of exponential(GARCH) models[6]. In (2022) investigate the studied of the stability study of exponential double autoregressive model[7]. In (2023) has been studied the stability conditions of limit cycle for Gompertz Autoregressive model[8]. The stability by using local linearization method was study in (2020)[9]. In (2023) has been studied the stability study of SATER models in a dynamic approach with application[10]. In (2023) has been studied stability conditions for a nonlinear time series model[11]. In (2024) has been suggestion a new analytical study of prey–predator dynamical systems involving the effects of Hide-and-Escape and predation skill augmentation[12]. This research aims to find the stability of conditions of a limit cycle for a nonlinear Lomax autoregressive model of order one, generalize these conditions of the order p, and provide several examples to elucidate stable or unstable orbits using plots of trajectories with different initial values.

2. CONCEPTS AND DEFINITIONS

The Lomax distribution, also known as the Pareto (II) distribution, is a heavy-tail probability distribution used in queuing theory, Internet traffic modelling, business, actuarial science, and economics, among other fields. It is named after[13]. it is essentially a Pareto distribution that has been shifted so that its support begins at zero. It shows the continuous probability distribution of a random variable that is not negative. The scale parameter is both a scale parameter and the distribution median. An indication of a shape parameter is ($\lambda > 0$).

The cumulative distribution function is given by:

$$F(x;\alpha,\lambda) = 1 - \left(1 + \frac{x_{t-1}}{\lambda}\right)^{-\alpha} \quad \dots (1)$$

Such that ($x > 0$, $\alpha > 0$, $\lambda > 0$).

Figure (1) it is clear plot of cumulative distribution function for Lomax distribution with different values of α and λ[14] .

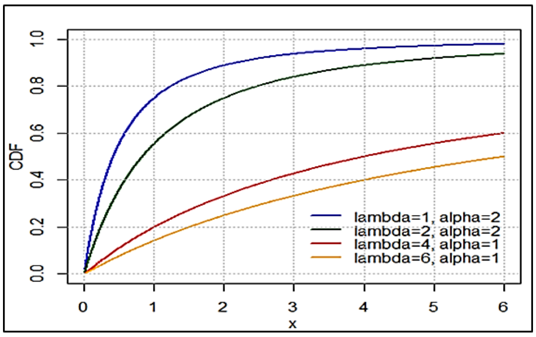

Fig. 1: Plotting the graph of the (c.d.f) Lomax distribution with different values of α and λ

2.1. Definition

The proposed model, Lomax AR (p), is defined by allowing $\{x_t\}$ to represent a discrete time series:

$$x_t = \sum_{i=1}^{p}\left[\mu_i + \beta_i\left[1 - \left(1 + \frac{x_{t-1}}{\lambda}\right)^{-\alpha}\right]\right] x_{t+1} + z_t \;, z_t \sim iidN(0, \sigma_z^2) \quad \dots (2)$$

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with $\{z_t\}$ representing a white noisiness process, α is scale parameter and $(\alpha > 0)$, λ is shape parameter $(\lambda > 0)$ and $\{\mu_i\}$, $\{\beta_i\}$ are serving as model constants and $(t = 1, 2, ..., p)$.

3. PRELIMINARIES

Below proposition (1) we find stability conditions of limit cycle, when Lomax autoregressive model of order 1 has a limit cycle of $(q > 1)$ where q is the period.

3.1. Proposition (1)

When following Lomax Autoregressive Model of order 1 has limit cycle of $(q > 1)$ where q is the period:

$$x_t = [\mu_1 + \beta_1 \left[1 - \left(1 + \frac{x_{t-1}}{\lambda}\right)^{-\alpha}\right] x_{t-1} + z_t] \quad \dots(3)$$

Then the model of equation (3) is orbital stable if and only if:

$$\left|\prod_{i=1}^{q}[\mu_1 + \beta_1\left(1 - \lambda^\alpha(\lambda + x_{t+q-i}\right)^{-\alpha} + \alpha\lambda^\alpha\left(\lambda + x_{t+q-i}\right)^{-\alpha-1}(x_{t+q-i})\right)]\right| < 1 \quad \dots(4)$$

3.1.1. Proof 1

First, we locate the model's non-zero singular point by using the definition of fixed point by putting $(x_t = x_{t-1} = x_{t-2} = \dots = x_{t-q} = l)$ after nigligate the effect of a white noise process $\{z_t\}$ where we assume that $(z_t = 0)$.

$$l = \sum_{i=1}^{p}\left[\mu_i + \beta_i\,(1 - (1 + \tfrac{l}{\lambda})^{-\alpha}\right] l$$

$$1 = \sum_{i=1}^{p}\left[\mu_i + \beta_i\,(1 - (1 + \tfrac{l}{\lambda})^{-\alpha}\right]$$

$$1 - \sum_{i=1}^{p}\mu_i = \sum_{i=1}^{p}\beta_i\,(1 - (1 + \tfrac{l}{\lambda})^{-\alpha}$$

$$1 - (1 + \tfrac{l}{\lambda})^{-\alpha} = \frac{1 - \sum_{i=1}^{p}\mu_i}{\sum_{i=1}^{p}\beta_i}$$

$$\text{Let } k = \frac{1 - \sum_{i=1}^{p}\mu_i}{\sum_{i=1}^{p}\beta_i} \quad \dots(5)$$

$$(1 + \tfrac{l}{\lambda})^{-\alpha} = 1 - k$$

$$1 + \frac{l}{\lambda} = (1 - k)^{-\frac{1}{\alpha}}$$

$$\frac{l}{\lambda} = (1 - k)^{-\frac{1}{\alpha}} - 1 \text{ therefore}$$

$$l = \lambda[(1 - k)^{-\frac{1}{\alpha}} - 1] \quad \dots(6)$$

of then non-zero singular point exists if and only if $1 - k > 0$

$$k < 1 \quad \text{or} \quad \frac{1 - \sum_{i=1}^{p}\mu_i}{\sum_{i=1}^{p}\beta_i} < 1$$

and suppose that the Lomax AR (1) possess a limit cycle of period $(q > 1)$ that is the limit cycle can be written as:

$$x_t, x_{t-1}, \dots, x_{t-q} = x_t \quad \dots(7)$$

near each point of a limit cycle with sufficiently small radius l_s of its neighborhood we use an assignment equation $(x_s = x_s + l_s)$ for $(s = t, t-1, t-2, ..., t-q)$ such that $(l_s^n \to 0$ for $n \geq 2)$ by substituting this assignment equation in the model, we get after niglicate a white noise process.

$$x_t + l_t = \left[\mu_1 + \beta_1[1 - \left(1 + \frac{x_{t-1} + l_{t-1}}{\lambda}\right)^{-\alpha}]\right](x_{t-1} + l_{t-1}) \quad \dots(8)$$

we simplify the term $(1 - \left(1 + \frac{x_{t-1} + l_{t-1}}{\lambda}\right)^{-\alpha})$ as follows:

$$1 - \lambda^\alpha(\lambda + x_{t-1} + l_{t-1})^{-\alpha} = 1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha}(1 + \frac{l_{t-1}}{\lambda + x_{t-1}})^{-\alpha}$$

And by using a Taylor expansion:

$$(1 + x)^{-k} = 1 - kx + \frac{k(k-1)}{2!}x^2 - \dots + (-1)^n \frac{k(k-1)\dots(k-n)}{n!}x^n$$

We get:

$$1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha}\left(1 + \frac{l_{t-1}}{\lambda + x_{t-1}}\right)^{-\alpha} = 1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha}(1 - \alpha\left(\frac{l_{t-1}}{\lambda + x_{t-1}}\right)) + \frac{\alpha(\alpha+1)}{2!}(\frac{l_{t-1}}{\lambda + x_{t-1}})^2$$

Since $(l_{t-1}^n \to 0$ for $n \geq 2)$

$$1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha}(1 - \alpha\frac{l_{t-1}}{\lambda + x_{t-1}}) = 1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha} + \alpha\lambda^\alpha(\lambda + x_{t-1})^{-\alpha}(\frac{l_{t-1}}{\lambda + x_{t-1}}) \quad \dots(3.7)$$

Substitute equation (8) in equation (7) we get

$$x_t + l_t = \left[\mu_1 + \beta_1[1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha} + \alpha\lambda^\alpha(\lambda + x_{t-1})^{-\alpha}(\frac{l_{t-1}}{\lambda + x_{t-1}})]\right](x_{t-1} + l_{t-1})$$

$$x_t + l_t = [\mu_1 + \beta_1\left[1 - \left(1 + \frac{x_{t-1}}{\lambda}\right)^{-\alpha}\right)]x_{t-1} + \alpha\lambda^\alpha(\lambda + x_{t-1})^{-\alpha-1}(x_{t-1})(l_{t-1})]$$
$$+ [\mu_1 + \beta_1[1 - \lambda^\alpha(\lambda + x_{t-1})^{-\alpha})](l_{t-1}) + \alpha\lambda^\alpha(\lambda + x_{t-1})^{-\alpha-1}(l_{t-1})(l_{t-1})$$

Since $(l_{t-1}^{n} \rightarrow 0$ for $n \geq 2)$

and since $x_t = \left[\mu_1 + \beta_1[1-\left(1+\frac{x_{t-1}}{\lambda}\right)^{-\alpha}]\right] x_{t-1}$

After suppressing the white noise process

$x_t + l_t = x_t + \alpha\lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha-1}(x_{t-1})(l_{t-1}) + [\mu_1 + \beta_1[1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha})](l_{t-1})$

$l_t = [\alpha\lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha-1}(x_{t-1}) + (\mu_1 + \beta_1[1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha}))]\,(l_{t-1})$

$l_t = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha}) + \alpha\lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha-1}(x_{t-1})\,]\,(l_{t-1})$ … (9)

This equation is difference equation with non-constant coefficient at its difficult to solve analytically but we can see the convergent condition by showing when the ratio $\left|\frac{l_{t+q}}{l_t}\right|$ approach to zero or not, that is $\left|\frac{l_{t+q}}{l_t}\right| < 1$

from (9) let $T(s) = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + s)^{-\alpha}) + \alpha\lambda^{\alpha}(\lambda + s)^{-\alpha-1}(s)\,]\,l_{t-1}$

where $s = x_{t-1}$

that is $l_t = T(x_{t-1})l_{t-1}$

and so on

$l_{t+1} = T(x_t)\,l_t = T(x_t)\,T(x_{t-1})\,l_{t-1} = T(x_t)\,T(x_{t-1})\,T(x_{t-2})\,l_{t-2}$

for $q > 1$ we get

$l_{t+q} = T(x_{t+q-1})\,l_{t+q-1} = T(x_{t+q-1})\,T(x_{t+q-2})\,l_{t+q-2}$ …

then $l_{t+q} = \prod_{i=1}^{q} T(x_{t+q-i})\,l_t$

$$\left|\frac{l_{t+q}}{l_t}\right| = \left|\prod_{i=1}^{q} T(x_{t+q-i})\right| \quad \dots (10)$$

then limit cycles of Lomax AR (1) model of order q are orbitally stable if and only if

$$\left|\prod_{i=1}^{q}[\mu_1 + \beta_1\left(1 - \lambda^{\alpha}(\lambda + x_{t+q-i})\right)^{-\alpha} + \alpha\lambda^{\alpha}(\lambda + x_{t+q-i})^{-\alpha-1}(x_{t+q-i})]\right| < 1$$

The following proposition is generalization of proposition (1) to Lomax AR (p) model.

3.2. Proposition 2

Limit cycles of period q for model Lomax AR(p)

$x_t = [\mu_1 + \beta_1\left[1 - \left(1 + \frac{x_{t-1}}{\lambda}\right)^{-\alpha}\right] x_{t-1} + z_t]$

stable if all eigen values of matrix

$R = R_q . R_{q-1} \dots\dots\dots . R_1$

have absolute values less than one

$$R_i = \begin{bmatrix} r_{1,1}^{(i)} & r_{1,2}^{(i)} & \cdots & r_{1,p-1}^{(i)} & r_{1,p}^{(i)} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \quad i=1,2,\dots,q$$

such that $r_{1,1}^{(i)} = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + x_{t+i-1})^{-\alpha}) + \sum_{j=1}^{p}\alpha\lambda^{\alpha}(\lambda + x_{t+i-j})^{-\alpha-1}(x_{t+i-j})$ … (11)

$r_{1,k}^{(i)} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_{t+i-1})^{-\alpha})$; $k = 2, 3, \dots, p$ … (12)

3.2.1. Proof 2

The Lomax AR (p) can be written after canceling the effect of white noise (z_t) where we assume that $(z_t = 0)$ in the state space form as

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-p+1} \end{bmatrix} = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,p-1} & m_{1,p} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p} \end{bmatrix}$$

… (13)

Such that $m_{1,k} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha})$; $k = 1, 2, 3, \dots, p$

Let the limit cycle with cycle q and q> 1 be the closed and isolated path known by equation (6) by taking an open neighborhood for each point of the limit cycle, that is by replacing each of x_s with $x_s + l$ for $s = t, t-1, t-2, \dots t-q$

Such that $l_s^{n} \rightarrow 0$ for $n \geq 2$

Therefor the equation (13) becomes the formula

$$x_t + l_t = \begin{bmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,p-1} & m_{1,p} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} (x_{t-1} + l_{t-1}) \quad \dots (14)$$

So that x_t , x_{t-1} , l_t , l_{t-1} defined on the state space R^p

$m_{1,k} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_{t+i-1})^{-\alpha})$; $k = 1, 2, 3, \dots, p$

And by conducting an approximation of the first and second terms of Taylor expansion for each element of the matrix and performing simple algebraic operations, we get:

$l_t = R_0\, l_{t-1}$ … (15)

So l_t , l_{t+1} defined on the state space R^p and R_0 it has the following formula

$$R_0 = \begin{bmatrix} r_{1,1}^{(0)} & r_{1,2}^{(0)} & \cdots & r_{1,p-1}^{(0)} & r_{1,p}^{(0)} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$r_{1,1}^{(0)} = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha}) + \alpha\lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha-1}(x_{t-1})$

$r_{1,k}^{(0)} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_{t-1})^{-\alpha})$; $k = 2, 3, \dots, p$

repeated the relationship (15) again, we get:

$l_{t+1} = R_1\, l_t$ … (16)

So

$$R_i = \begin{bmatrix} r_{1,1}^{(i)} & r_{1,2}^{(i)} & \cdots & r_{1,p-1}^{(i)} & r_{1,p}^{(i)} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

Such that $i = 1, 2, \dots, q$ and

$r_{1,1}^{(1)} = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + x_t)^{-\alpha}) + \alpha\lambda^{\alpha}(\lambda + x_t)^{-\alpha-1}(x_t)$

$r_{1,k}^{(1)} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_t)^{-\alpha})$; $k = 2, 3, \dots, p$

Repeated the relationship (16) q-times, we get:

$l_{t+q} = R_q\, l_{t+q-1} = R_q R_{q-1} l_{t+q-2} \cdots$

$l_{t+q} = \prod_{i=1}^{q} R_i \,.\, l_t$ … (17)

Let R be the product of the matrices R_i for i=1,2, 3,…q the equation (17) can be written

$l_{t+q} = R\, l_t$ … (18)

for convergence of R_j to zero as j→∞ then the absolute values of the eigenvalues of the matrix R lies inside the unite circle

such that $R = \prod_{i=1}^{q} R_i$ and

$$R_i = \begin{bmatrix} r_{1,1}^{(i)} & r_{1,2}^{(i)} & \cdots & r_{1,p-1}^{(i)} & r_{1,p}^{(i)} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \text{ i=1, 2,…q}$$

Such that

$r_{1,1}^{(i)} = [\mu_1 + \beta_1(1 - \lambda^{\alpha}(\lambda + x_{t+i-1})^{-\alpha}) + \sum_{j=1}^{p} \alpha\lambda^{\alpha}(\lambda + x_{t+i-j})^{-\alpha-1}(x_{t+i-j})$

$r_{1,k}^{(i)} = \mu_k + \beta_k(1 - \lambda^{\alpha}(\lambda + x_{t+i-1})^{-\alpha})$; $k = 2, 3, \dots, p$

Then the Lomax AR (p) is orbitally stability if all the eigen values of a matrix R has absolute values less than 1.

4. APPLICATION

In this paragraph, we will apply the conditions for stability of the limit cycle in proposition (1) by taking some examples of stable and unstable limit cycles. And we will utilize our work with the program MATLAB to find the periods of limit cycle.

4.1. Example 1

Consider the Lomax AR (1) model is given by

$$x_t = \left[-1.45 + 2.64[1 - \left(1 + \frac{x_{t-1}}{1.4}\right)^{-2.1}]\right] x_{t-1} + z_t$$

has a limit cycle of period 2 which is $\{0.3559, -0.1604\}$ look at appendix. we can calculate it by using the following condition of the equation (3).

$$\left|\prod_{i=1}^{2}[-1.45 + 2.64(1 - 1.4^{2.1}(1.4 + x_{t+i-1})^{-2.1}) + (2.1)(1.4)^{2.1}(1.4 + x_{t+q-i})^{-2.1-1}(x_{t+q-i})]\right| = 0.7273 < 1$$

Since the condition of equation (3) is satisfied, the limit cycle is orbitally stable. The stability of a limit cycle with various beginning values is displayed in Figure (2).

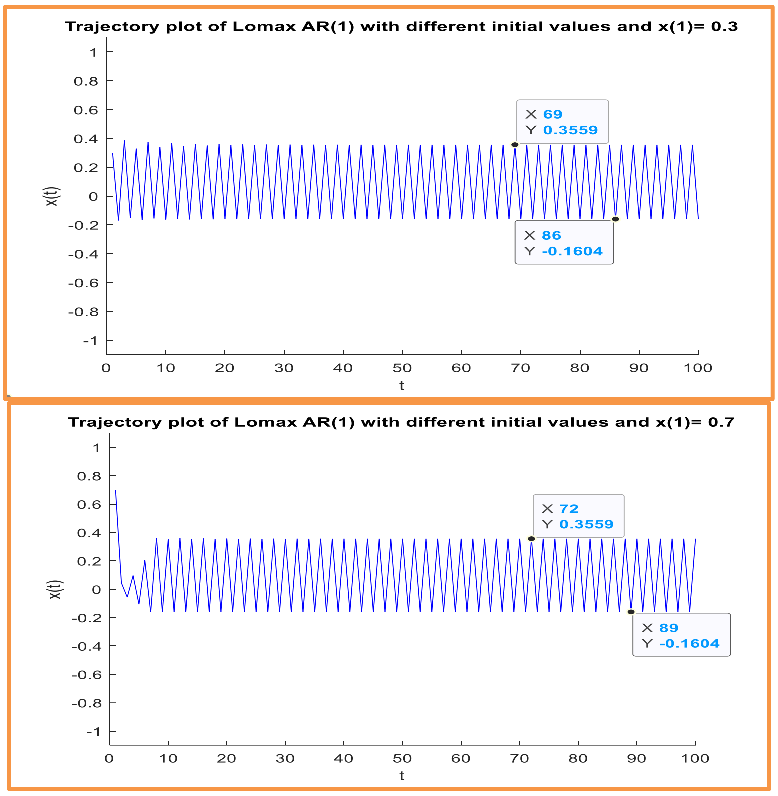

Fig. 2: The orbitally stable for different starting values

$$\left|\prod_{i=1}^{4}[-1.75 + 2.99(1 - 1.1^{2.1}(1.1 + x_{t+i-1})^{-2.1}) + (2.1)(1.1)^{2.1}(1.1 + x_{t+q-i})^{-2.1-1}(x_{t+q-i})]\right| = 3.5529 > 1$$

the limit cycle is orbitally unstable and does not to satisfy the condition (3). take notice of the varied starting values in Figure (3).

4.2. Example 2

Let the following be Lomax AR (1) model is

$$x_t = \left[-1.75 + 2.99[1 - \left(1 + \frac{x_{t-1}}{1.1}\right)^{-2.1}]\right] x_{t-1} + z_t$$

having a limit cycle of period 4 that is $\{0.54, -0.08, 0.18, -0.18, 0.54\}$, and we may compute it by using the subsequent equation condition (3).

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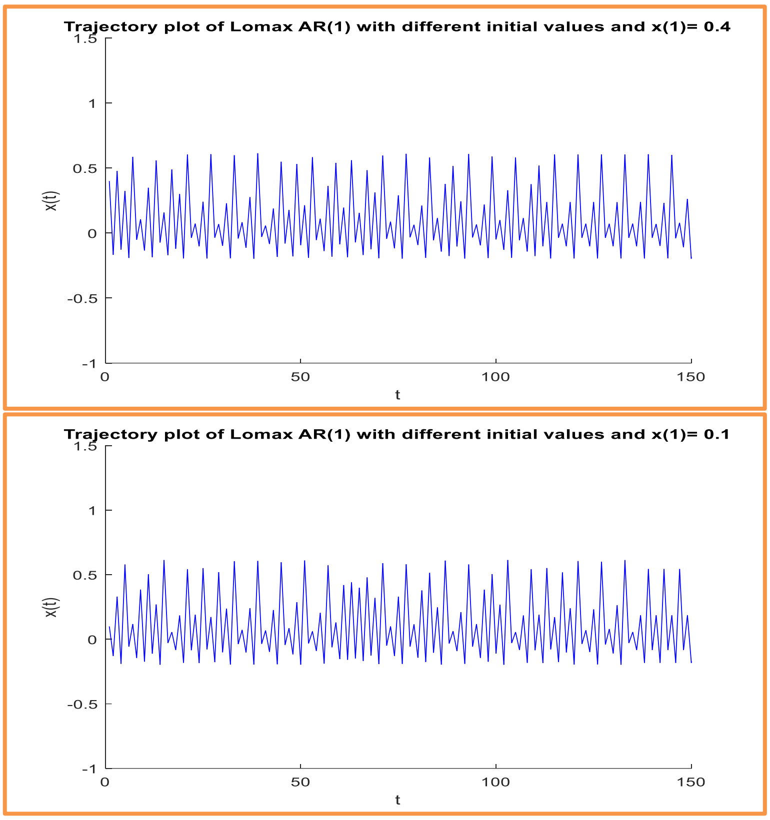

Fig. 3: The orbitally unstable for different initial values

5. CONCLUSION

In this research, we found stability conditions for the limit cycle of the Lomax autoregressive model of order one as in proposition (1). We generalized these conditions to the Lomax autoregressive model of order p and represented it on state space as in proposition (2). We used the local linearization approximation technique to prove this condition, and then we explained this condition by giving some examples and plot trajectories that illustrate stability.

Appendix

The algorithm used in MATLAB for examples

```
clear; clc; a1=-1.45; b1=2.64; c=1.4; d=2.1; a=a1;b=b1;
k=(1-a)/b; f=c*(((1-k) ^(-1/d))-1); xlabel('t'),ylabel('x(t)');
title ('Trajectory plot of Lomax AR (1) with different initial values and x (1) = 0.7');
axis ([0 100 -1.1 1.1]);
N=394; number.data=N; K=zeros (1,300);
for j=1:1
  K(j)=0.7;
end
for i=2:300
  K(i)=a1*K(i-1) +(b1*K(i-1)) *(1-(1+(K(i-1)/c))^(-d));
    hold on
      plot([(i), (i-1)], [K(i), K(i-1)],'b');
    hold off
end
 disp(f);disp(K);
```

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Author contribution: Author contributed in the study.

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