



The Concircular Curvature Tensor Of The Locally Conformal Kahler Manifold

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ABSTRACT

In this research, we are calculated components conharmonic curvature tensor in some aspects Hermeation manifold in particular of the Locally Conformal Kahler manifold. And we prove that this tensor possesses the classical symmetry properties of the Riemannian curvature. They also, establish relationships between the components of the tensor in this manifold.

1- Introduction

Concircular curvature tensor is invariant under concircular transformations, i.e. with conformal transformations of space keeping a harmony of functions. The concircular curvature tensor introducer will be Reminded Yano on 1940 as a tensor of type (4, 0) on n-dimensional Riemannian manifold, Conformal transformations of Riemannien structures are the important object of differential geometry, Rawah A.Z. Hassan on 2015 researched concirculac curvature tensor of nearly Kahler manifold, in this paper we investigate the "concirculac curvature tensor of locally conformal Kahler manifold".

2- Preliminaries

Let M –"smooth manifold of dimension 2n", the concircular curvature tensor introducer will be Reminded Yano as a tensor of type (4, 0) on n-dimensional Riemannian manifold. An AH -manifold is called a locally conformal Kahler manifold, if foreach point $m \in M$ there exist an open neighborhood U of this point and there exists $f \in C^\infty(M)$ such that \tilde{U}_f is Kahler manifold [2]. We will denoted to the locally conformal Kahler manifold by L.C.K.

Definition 1.1 [2]

An AH-manifold is called a locally conformal Kahler manifold, if foreach point $m \in M$,there exist an open neighborhood U of this point and there exists

$f \in C^\infty(M)$ such that \tilde{U}_f is Kahler manifold . We will denoted to the locally conformal Kahler manifold by L.C.K .

Remark 1.2 [3]

By the Banaru's classification of AH-manifold, the L.C.K- manifold satisfies the following conditions : $B^{abc} = 0$, $B_c^{ab} = \alpha^{[a}\delta_c^{b]}$, where B^{abc} and B_c^{ab} system of function on M .

Theorem 1.3 [4]

The structure equations of L.C.K- manifold in the adjoint G – structure space is given by the following forms :

1. $d\omega^a = \omega_b^a \wedge \omega^b + B_c^{ab} \omega^c \wedge \omega_b$
2. $d\omega_a = -\omega_a^b \wedge \omega_b + B_{ab}^c \omega_c \wedge \omega^b$
3. $d\omega_b^a = \omega_c^a \wedge \omega_b^c + A_{bc}^{ad} \omega^c \wedge \omega_d + \{\frac{1}{2}\alpha^{a[c}\delta_b^{d]} + \frac{1}{4}\alpha^a\alpha^{[c}\delta_b^{d]}\}\omega_c \wedge \omega_d$

Theorem 1.4 [4]

In the adjoint G –structure spaace , the component of Riemannian curvature tensor of L.C.K- manifold are given by the following forms :

1. $R_{bcd}^a = \alpha_{a[c}\delta_{d]}^b + \frac{1}{2}\alpha_a\alpha_{[c}\delta_{d]}^b$
2. $R_{\hat{b}\hat{c}\hat{d}}^{\hat{a}} = -\alpha^{a[c}\delta_{b]}^d - \frac{1}{2}\alpha^a\alpha^{[c}\delta_{b]}^d$
3. $R_{\hat{b}\hat{c}d}^{\hat{a}} = -2\alpha_{[c}^{[a}\delta_{d]}^b$
4. $R_{\hat{b}\hat{c}\hat{d}}^{\hat{a}} = 2\alpha_{[a}^{[c}\delta_{b]}^d$
5. $R_{bcd}^a = A_{bc}^{ad} - \alpha^{[a}\delta_c^{h]}\alpha_{[h}\delta_b^d$

We shall prove just (1) the rest is as proof in the same way

$$\begin{aligned} 1) \quad C(LCK)(X_a, X_b, X_c, X_d) &= R(X_a, X_b, X_c, X_d) - \\ &\frac{\chi}{n(n-1)} \{g(X_a, X_c)r(X_b, X_d) - g(X_b, X_c)r(X_a, X_d)\} \\ &= -R(X_a, X_b, X_c, X_d) + \frac{\chi}{n(n-1)} \{g(X_a, X_c)r(X_b, X_d) - \\ &g(X_b, X_c)r(X_a, X_d)\} = -R(X_b, X_a, X_c, X_d) \end{aligned}$$

Properties are similarly proved:

$$\begin{aligned} 2) C(LCK)(X_a, X_b, X_c, X_d) &= -R(X_a, X_b, X_d, X_c) \\ 3) \quad C(LCK)(X_a, X_b, X_c, X_d) + R(X_b, X_a, X_c, X_d) + \\ R(X_c, X_a, X_b, X_d) &= 0 \\ 4) \quad C(LCK)(X_a, X_b, X_c, X_d) &= -R(X_c, X_d, X_a, X_b) \end{aligned}$$

$X_i \in X(M)$, $i = 1, 2, 3, 4$

(1),(2),(3) and (4) is called an algebra curvature tensor of (L.C.K) manifolds.

The cocircular curvature of (L.C.K) manifolds looks like

$$R(X_a, X_b)X_c = R(X_a, X_b)X_c - \frac{\chi}{n(n-1)} \{ < X_b, X_c > \\ X_a r - < X_a, X_c > QX_b \}$$

Where $Q = r$.

By definition of a spectrum tensor.

$$\begin{aligned} R(X_a, X_b)X_c &= R_0 < X_a, X_b > X_c + R_1 < X_a, X_b > \\ &X_c + R_2 < X_a, X_b > X_c + R_3 < X_a, X_b > X_c + R_4 < \\ &X_a, X_b > X_c + R_5 < X_a, X_b > X_c + R_6 < X_a, X_b > \\ &X_c + R_7 < X_a, X_b > X_c \end{aligned}$$

Tensor $R_0 < X_a, X_b > X_c$ nonzero – the component have only components of the form :

$$\begin{aligned} \text{Tensor} \quad C_0(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_0^a{}_{bcd}(LCK), C_0^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_1(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_1^a{}_{bcd}(LCK), C_1^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_2(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_2^a{}_{bcd}(LCK), C_2^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_3(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_3^a{}_{bcd}(LCK), C_3^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_4(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_4^a{}_{bcd}(LCK), C_4^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_5(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_5^a{}_{bcd}(LCK), C_5^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}(LCK)\} &= \\ \{C_{bc\hat{d}}^a(LCK), C_{\hat{b}\hat{c}\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_6(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_6^a{}_{\hat{b}\hat{c}\hat{d}}(LCK), C_6^{\hat{a}}{}_{bc\hat{d}}(LCK)\} &= \\ \{C_{\hat{b}\hat{c}\hat{d}}^a(LCK), C_{bc\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensor} \quad C_7(LCK) &< X_a, X_b > \\ X_c - \text{components} \{C_7^a{}_{\hat{b}\hat{c}\hat{d}}(LCK), C_7^{\hat{a}}{}_{bc\hat{d}}(LCK)\} &= \\ \{C_{\hat{b}\hat{c}\hat{d}}^a(LCK), C_{bc\hat{d}}^{\hat{a}}(LCK)\} \end{aligned}$$

$$\begin{aligned} \text{Tensors} \quad R_0 &= R_0 < X_a, X_b > X_c, R_1 = R_1 < \\ X_a, X_b > X_c, \dots, R_7 &= R_7 < X_a, X_b > X_c. \end{aligned}$$

The basic invariants cocircular (L.C.K) manifold will be named.

Definition 1.11

LCK- manifold for which $C_i(LCK) = 0$ is LCK-manifold of class $C_i(LCK)$, $i = 0, 1, \dots, 7$.

The manifold of class $C_0(LCK)$ characterized by a condition $C_0^a{}_{bcd}(LCK) = 0$, or

$C^a{}_{bcd} = 0$, $[C(LCK)(\varepsilon_c, \varepsilon_d)\varepsilon_b]^a\varepsilon_a = 0$. As σ - a projector on $D^{Y^{-1}}$, that

$$\sigma \circ \{C(LCK)(\sigma X_a, \sigma X_b)\sigma X_c = 0,$$

$$\text{i.e. } (id - \sqrt{-1}J)\{C(LCK)(X - \sqrt{-1}JX, Y - \sqrt{-1}JY)(Z - \sqrt{-1}JZ)\} = 0.$$

Removing the brackets can be received :

$$\begin{aligned} C(LCK)(X_a, X_b)X_c &- C(LCK)(X_a, JX_b)JX_c - \\ C(LCK)(JX_a, X_b)JX_c &- C(LCK)(JX_a, JX_b)X_c - \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &+ JC(LCK)(JX_a, JX_b)JX_c - \\ \sqrt{-1}\{C(LCK)(X_a, X_b)JX_c + C(LCK)(X_a, JX_b)X_c + \\ C(LCK)(JX_a, X_b)X_c &- C(LCK)(JX_a, JX_b)JX_c\} - \\ JC(LCK)(X_a, X_b)X_c &- JC(LCK)(X_a, JX_b)JX_c - \\ JC(LCK)(JX_a, X_b)JX_c &+ JC(LCK)(JX_a, JX_b)X_c\} = 0, \\ \text{i.e. 1) } \quad C(LCK)(X_a, X_b)X_c &- C(LCK)(X_a, JX_b)JX_c - \\ C(LCK)(JX_a, X_b)JX_c &- C(LCK)(JX_a, JX_b)X_c - \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &+ JC(LCK)(JX_a, JX_b)JX_c = 0, \end{aligned}$$

Thus LCK- manifold of class $C_0(LCK)$ characterized by identity

$$\begin{aligned} 2) \quad C(LCK)(X_a, X_b)X_c &+ C(LCK)(X_a, JX_b)JX_c - \\ C(LCK)(JX_a, X_b)JX_c &+ C(LCK)(JX_a, JX_b)X_c + \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &- JC(LCK)(JX_a, JX_b)JX_c = \\ 0, X_a, X_b, X_c \in X(M). \end{aligned}$$

These equalities are equivalent . The second equality turns out from the first

Replacement Z on JZ .

$$\begin{aligned} C(LCK)(X_a, X_b)X_c &- C(LCK)(X_a, JX_b)JX_c - \\ C(LCK)(JX_a, X_b)JX_c &- C(LCK)(JX_a, JX_b)X_c - \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &+ JC(LCK)(JX_a, JX_b)JX_c = \\ 0, X_a, X_b, X_c \in X(M). \end{aligned}$$

Similary considering LCK- manifold of classes $R_1 - R_7$ can be receved the following theorem .

Theorem 1.12

1) LCK- manifold of class $C_0(LCK)$ characterized by identity

$$\begin{aligned} C(LCK)(X_a, X_b)X_c &- C(LCK)(X_a, JX_b)JX_a - \\ C(LCK)(JX_a, X_b)JX_c &- C(LCK)(JX_a, JX_b)X_c - \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &+ JC(LCK)(JX_a, JX_b)JX_c = \\ 0, X_a, X_b, X_c \in X(M). \end{aligned}$$

2) LCK- manifold of class $C_1(LCK)$ characterized by identity

$$\begin{aligned} C(LCK)(X_a, X_b)X_c &+ C(LCK)(X_a, JX_b)JX_c - \\ C(LCK)(JX_a, X_b)JX_c &+ C(LCK)(JX_a, JX_b)X_c + \\ JC(LCK)(X_a, X_b)JX_c &- JC(LCK)(X_a, JX_b)X_c - \\ JC(LCK)(JX_a, X_b)X_c &- JC(LCK)(JX_a, JX_b)JX_c = \\ 0, X_a, X_b, X_c \in X(M). \end{aligned}$$

3) LCK – manifold of class $C_2(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c - JC(LCK)(X_a, JX_b)X_c + \\ & JC(LCK)(JX_a, X_b)X_c - JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

4) $LCK -$

manifold of class $C_3(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c - C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c + JC(LCK)(X_a, JX_b)X_c + \\ & JC(LCK)(JX_a, X_b)X_c + JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

5) $LCK -$ manifold of class $C_4(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c - C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c - JC(LCK)(X_a, JX_b)X_c - \\ & JC(LCK)(JX_a, X_b)X_c - JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

6) $LCK -$ manifold of class $C_5(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c + JC(LCK)(X_a, JX_b)X_c - \\ & JC(LCK)(JX_a, X_b)X_c + JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

7) $LCK -$

manifold of class $C_6(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c + C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c - JC(LCK)(X_a, JX_b)X_c + \\ & JC(LCK)(JX_a, X_b)X_c + JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

8) $LCK -$

manifold of class $C_7(LCK)$ characterized by identity

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c - C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c + JC(LCK)(X_a, JX_b)X_c + \\ & JC(LCK)(JX_a, X_b)X_c - JC(LCK)(JX_a, JX_b)JX_c = \\ & 0, \quad X_a, X_b, X_c \in X(M). \end{aligned}$$

Definition 1.13

The manifold (M, J, g) refers to as manifold of a class:

1) $\bar{C}_1(LCK)$ if $\langle C(LCK)(X, Y)Z, W \rangle = \langle C(LCK)(X, Y)JZ, JW \rangle$;

2) $\bar{C}_2(LCK)$ if $\langle C(LCK)(X, Y)Z, W \rangle = \langle C(LCK)(JX, JY)Z, W \rangle + \langle C(LCK)(JX, Y)JZ, W \rangle + \langle C(LCK)(JX, Y)Z, JW \rangle$;

3) $\bar{C}_3(LCK)$ if $\langle C(LCK)(X, Y)Z, W \rangle = \langle C(LCK)(JX, JY)JZ, JW \rangle$

Theorem 1.14

Studying and creating some of the relationships involved:

i) $C_0(LCK) = C_3(LCK)$;

ii) $C_1(LCK) = C_2(LCK)$;

iii) $C_4(LCK) = C_7(LCK)$;

iv) $C_5(LCK) = C_6(LCK)$.

Proof: - We shall prove (i)

$$\begin{aligned} & C_0(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c - JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (1) \end{aligned}$$

$$\begin{aligned} & C_0(LCK) = \\ & C(LCK)(X_a, X_b)X_c - \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c - \\ & + (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \end{aligned}$$

$$\begin{aligned} & C_0(LCK) = \\ & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c - JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (2) \end{aligned}$$

From (1) and (2) we get

$$\begin{aligned} & C_0(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c + JC(LCK)(JX_a, JX_b)JX_c \dots (3) \end{aligned}$$

$$\begin{aligned} & C_3(LCK) = \\ & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c - J(LCK)C(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (4) \end{aligned}$$

$$\begin{aligned} & C_3(LCK) = \\ & C(LCK)(X_a, X_b)X_c + \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \end{aligned}$$

$$\begin{aligned} & C_3(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c - JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(X_a, JX_b)X_c + JC(JX_a, JX_b)JX_c \dots (5) \end{aligned}$$

From (4) and (5) we get

$$\begin{aligned} & C_3(LCK) = C(X_a, X_b)X_c - C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (6) \end{aligned}$$

From (3) and (6) we get $C_0(LCK) = C_3(LCK)$

Now we shall prove (ii)

$$\begin{aligned} & C_1(LCK) = \\ & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c + \end{aligned}$$

$$C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c - JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c - JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (7)$$

$C_1(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \end{aligned}$$

$C_1(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + \\ & C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (8) \end{aligned}$$

From (7)and (8) we get

$$\begin{aligned} & C_1(LCK) = \\ & C(LCK)(X_a, X_b)X_c + C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c - JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (9) \end{aligned}$$

$C_2(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + \\ & C(LCK)(JX_a, JX_b)X_c - JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (10) \end{aligned}$$

$C_2(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c - \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \end{aligned}$$

$C_2(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c + \\ & C(LCK)(JX_a, JX_b)X_c - JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (11) \end{aligned}$$

From (10)and (11) we get

$$\begin{aligned} & C_2(LCK) = \\ & C(LCK)(X_a, X_b)X_c + C(LCK)(JX_a, JX_b)X_c - \\ & JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (12) \end{aligned}$$

From (9) and (12) we get $C_1(LCK) = C_2(LCK)$

Now we shall prove (iii)

$C_4(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c - C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c - JC(LCK)(X_a, JX_b)X_c - \\ & JC(LCK)(JX_a, X_b)X_c - JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (13) \end{aligned}$$

$C_4(LCK) =$

$$\begin{aligned} & C(LCK)(X_a, X_b)X_c + \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c + \end{aligned}$$

$$\begin{aligned} & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c - \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \\ & C_4(LCK) = C(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (14) \end{aligned}$$

From (13)and (14) we get

$$\begin{aligned} & C_4(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (15) \end{aligned}$$

$$\begin{aligned} & C_7(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (16) \end{aligned}$$

$$\begin{aligned} & C_7(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c - \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \\ & C_7(LCK) = C(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c + C(LCK)(JX_a, X_b)JX_c - \\ & C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c - \\ & JC(LCK)(JX_1, JX_2)JX_3 \dots \dots \dots (17) \end{aligned}$$

From (16)and (17) we get

$$\begin{aligned} & C_7(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (18) \end{aligned}$$

From (15) and (18) we get $C_4(LCK) = C_7(LCK)$

Now we shall prove (iv)

$$\begin{aligned} & C_5(LCK) = \\ & C(LCK)(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (19) \end{aligned}$$

$$\begin{aligned} & C_5(LCK) = \\ & C(LCK)(X_a, X_b)X_c - \\ & C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c + \\ & C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c + \\ & (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c - \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c + \\ & (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \\ & C_5(LCK) = C(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c + \end{aligned}$$

$$\begin{aligned} C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c - \\ JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c + \\ JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (20) \end{aligned}$$

From (19)and (20) we get

$$\begin{aligned} C_5(LCK) = & C(LCK)(X_a, X_b)X_c + C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (21) \\ C_6(LCK) = & C(LCK)(X_a, X_b)X_c + C(LCK)(X_a, JX_b)JX_c - \\ & C(LCK)(JX_a, X_b)JX_c \\ & + C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c - \\ & JC(LCK)(X_a, JX_b)X_c + JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (22) \end{aligned}$$

$$\begin{aligned} C_6(LCK) = & C(X_a, X_b)X_c + C(LCK)(X_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \\ & - C(LCK)(-\sqrt{-1}JX_a, X_b)(-\sqrt{-1})JX_c \\ & + C(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)X_c \\ & + (-\sqrt{-1})JC(LCK)(X_a, X_b)(-\sqrt{-1})JX_c \\ & - (-\sqrt{-1})JC(LCK)(X_a, -\sqrt{-1}JX_b)X_c \\ & + (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, X_b)X_c \\ & + (-\sqrt{-1})JC(LCK)(-\sqrt{-1}JX_a, -\sqrt{-1}JX_b)(-\sqrt{-1})JX_c \\ C_6(LCK) = & C(X_a, X_b)X_c - C(LCK)(X_a, JX_b)JX_c + \\ & C(LCK)(JX_a, X_b)JX_c + \\ & C(LCK)(JX_a, JX_b)X_c + JC(LCK)(X_a, X_b)JX_c + \\ & JC(LCK)(X_a, JX_b)X_c - JC(LCK)(JX_a, X_b)X_c + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots \dots \dots (23) \end{aligned}$$

From (22)and (23) we get

$$\begin{aligned} C_6(LCK) = & C(LCK)(X_a, X_b)X_c + C(LCK)(JX_a, JX_b)X_c + \\ & JC(LCK)(X_a, X_b)JX_b + \\ & JC(LCK)(JX_a, JX_b)JX_c \dots (24) \end{aligned}$$

From (21) and (24) we get $C_5(LCK) = C_6(LCK)$

Theorem 1.15

Let $S = (J, g = \langle x, x \rangle)$ – is L.C.K. then the following statement are equivalent :

- 1) S – structure of class $\bar{C}_3(LCK)$
- 2) $C_7(LCK) = 0$
- 3) On space of the adjoint G – structure identities $C(LCK)^{\hat{a}}_{bcd} = 0$ are fair .

Proof:

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Let S – structure of class $\bar{C}_3(LCK)$. Obviously it is equivalent to identity

$C(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + JC(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c = 0 ; \varepsilon_a, \varepsilon_b, \varepsilon_c \in X(M)$. By definition of a spectrum tensor

$$\begin{aligned} C(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c &= C_0(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + \\ C_1(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c &+ C_2(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + \\ C_3(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c &+ C_4(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + \\ C_5(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c &+ C_6(LCK)(\varepsilon_a, \varepsilon_b)X_c + \\ C_7(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c, & \varepsilon_a, \varepsilon_b, \varepsilon_c \in X(M) \\ JC(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c &= JC_0(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c + \\ JC_1(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c &+ JC_2(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c + \\ JC_3(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c &+ JC_4(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c + \\ JC_5(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c &+ JC_6(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c + \\ JC_7(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c, & \varepsilon_a, \varepsilon_b, \varepsilon_c \in X(M) \end{aligned}$$

The identity

$$C(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + C(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c = 0 \text{ is equivalent to that } C_7(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + C_4(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + C_5(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + C_6(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c = 0$$

And this is equivalent to identities $C_7(LCK) = C_4(LCK) = C_5(LCK) = C_8(LCK) = 0$

By virtue of materiality tensor $C(LCK)$ and its properties (3.2.6) received relation which are equivalent to relations $C_7(LCK)^{\hat{a}}_{bcd} = 0$, i.e.identity $C_7(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c = 0$

The opposite , according to

$$C(LCK)(\varepsilon_a, \varepsilon_b)\varepsilon_c + JC(LCK)(J\varepsilon_a, J\varepsilon_b)J\varepsilon_c = 0 ; \varepsilon_a, \varepsilon_b, \varepsilon_c \in X(M). \text{obviously.}$$

Conclusion

The main results of this study are stated below :

- 1) Computing components of this tensor which are $C_0(L.C.K), C_1(L.C.K), C_2(L.C.K), C_3(L.C.K), C_4(L.C.K), C_5(L.C.K), C_6(L.C.K), C_7(L.C.K)$ in Locally Conformal Kahler Manifold (L.C.K).

- 2) Neutral equations for these component $C_0(L.C.K), C_1(L.C.K), C_2(L.C.K), \dots, C_7(L.C.K)$ Almost Hermation Manifold.

- 3) Find new classes $\bar{C}_0(L.C.K), \bar{C}_1(L.C.K)$ and $\bar{C}_3(L.C.K)$ and proved the structure $\bar{C}_3(L.C.K)$ is $C_7(L.C.K) = 0$, and on space of the adjoint G -structure identities $C(L.C.K)^{\hat{a}}_{\hat{b}\hat{c}\hat{d}} = 0$ are fair.

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[6] Meleva p " Locally conformal Kahler Manifold of constant tipe And J-invariant Curvature tensor " Facta Universitatis, Series Mechanacs, Automatic control and Robotics Vo1.3,no,14,Pp,791.804,2003.

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تنسir الانحناء الدائري في منطوي كوهنر المتطابق محليا

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الملخص

في هذا البحث تم احتساب مركبات تنسير الانحناء الدائري في بعض أصناف المنطوي الهرمي التقربي وعلى وجهه الخصوص منطوي كوهنر الكونفورمي المحلي، مع برهنة ان هذا التنzer يمتلك خصائص التناظر الكلاسيكي لتنzer الانحناء الريمانى، إضافة الى ايجاد علاقات بين مركبات التنzer في هذا المنطوي.