

# Controllability of solutions for semilinear fractional integrodifferential equations in Banach spaces

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## Abstract

The study aims to prove the controllability of mild solution for semilinear fractional integrodifferential equations with nonlocal conditions in Banach spaces.

Fractional calculus, compact semigroups and fixed point theorem, are concepts consulted to obtain results this work.

## Introduction

Studying fractional differential equations have been contronersial nowadays , for type of these equations are arised in many scopes of applied mathematics and seientific disciplines such as chemistry, biology, blood flow phenomena, control theory, etc, see [1,2]. There has been noticeable development in ordinary and partial differential equations involving Riemann-liouville fractional derivatives, for details see [3].

Many researchers have been also extensively studied controllability problems of linear and nonlinear systems reflected in ordinary differential equations in finite dimensional spaces, see [4].

The aim of the work is to study the controllability of fractional for semilinear in tegrodifferential systems with nonlocal condition in Banach spaces.

The main idea used to study the controllability result for the problem (3.1) is to prove that the operator "P" defined by (3.2) is completely continuous in  $C(o, b; X)$  which guarantee the fixed point theorem of schauder, for this purpose the work is organized according to preliminaries from the fields of fractional integral and derivative maps. Finally, we build up the controllability result for the system (3.1), and this result is a generalization of the problem in [5].

## 1- Preliminaries:

To study and complete this work we need concepts and necessary basic.

At first, let  $X$  is areal Banach space with norm  $\| \cdot \|$ , the space of  $X$ -valued continuous functions on  $[0, b]$  denoted by  $([0, b]; X)$  and the space of  $X$ -valued Bochner integrable functions on  $[0, b]$  denoted by  $L(0, b; X)$ .

### Definition (2.1), [6]:

"Let  $X=(x, d)$  and  $Y=(y, d)$  be ametric spaces, A mapping  $T: X \rightarrow Y$  is said to be continuous at a point  $x_0 \in X$  if for every  $\epsilon > 0$  there is a  $\delta > 0$  such that  $d(Tx, Tx_0) < \epsilon$  for all  $x$  satisfying  $d(x, x_0) < \delta$  and  $T$  is said be continuous if it is continuous at every point of  $X$ ".

### Definition (2.2), [7]:

"Let  $M$  be a subset of normed space  $X$ . Then the intersection of all convex subset of  $X$  containing  $M$  is the smallest convex subset of  $X$  containing  $M$ , this called the convex hull or convex envelope of  $M$  and is usually denoted by  $com, com = \bigcap \{A_i | A_i \text{ convex and } M \subseteq A_i\}$ . The closure of the

convex hull  $M$ , that is  $\overline{com}$  is called the closed convex hull of  $M$ ".

### Definition (2.3) [8]:

"For a function  $f$  given on the fractional derivative of  $f$  is defined by

$$(D_a^\alpha f)(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-s)^{n-\alpha-1} f(s) ds, n-1 < \alpha < n$$

### Definition (2.4), [8]

"The functional (arbitrary) order integral of the function

$f \in L^1([a, b], R^+)$  of order  $\alpha \in R^+$  is defined by

$$I_a^\alpha f(t) = \int_a^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f(s) ds, \text{ where } \Gamma \text{ is the gamma function}."$$

### Definition (2.5), [9]

"Let  $X$  and  $Y$  be normed spaces, the operator  $A: D(A) \subseteq X \rightarrow Y$  is said to be compact : if

1.  $A$  is continuous.
2.  $A$  is transforms every bounded subset  $M$  of  $X$  into relatively compact subset of  $Y$  ( $\overline{A(M)}$  is compact) "

### Definition (2.6), [10]:

"A subset  $U$  of  $C[a, b]$  is said to be equicontinuous if for each  $\epsilon > 0$ , there is  $\delta > 0$  such that  $\|x - y\| < \delta$  and  $u \in U$ , imply  $\|u(x) - u(y)\| < \epsilon$ ."

### Remark (2.7), [6]:

"A map  $K: X \rightarrow 2^X$   $\{\phi\}$  is convex (closed valued) if  $K(x)$  is convex (closed) for all  $x \in X$ .  $K$  is bounded into bounded sets if  $K([0, b]) = \bigcup_{x \in [0, b]} K(x)$  is bounded in  $X$  for any bounded set  $[0, b]$  of  $X$ ".

### Definition (2.8), [6]:

"A map  $K$  is called completely continuous if  $K([0, b])$  is relatively compact for every bounded subset  $[0, b]$  of  $X$ ".

### Theorem (2.9), (Arzela - Ascoli's theorem), [5]:

"Let  $F \subset C([0, b]; X)$  satisfy:

- i) For any  $t \in [a, b]$ ,  $\{f(t): f \in F\}$  is relatively compact in  $X$ .
- ii)  $F$  is equicontinuous on  $[a, b]$ , that is, for any  $\epsilon > 0$  and any  $t \in [a, b]$ , there exit  $\delta > 0$  such that,  $\|f(t) - f(s)\| < \epsilon$ , for any  $\delta \in [a, b]$  satisfying  $|t - s| < \delta$ , and all  $f \in F$ . Then  $F$  is relatively compact".

### Theorem (2.10), [5], (Schauder's Fixed-point theorem)

"Let  $C$  be a nonempty bounded convex closed subset in  $X$ , if  $F: C \rightarrow C$  is continuous and  $F(C)$  is relatively compact, then  $F$  has at least one fixed point".

### 3. Controllability results

In this section, given the system,

$$D^\alpha(z(t)) = \mathcal{R}z(t) + Bu(t) + f(t, z(t)),$$

$$t \in [0, b], 0 < \alpha < 1$$

$$Z(0) = Z_0 \in X \quad (3.1)$$

Is integrodifferential semilinear fractional Where  $\mathcal{R}(t)$  is bounded linear operator strongly continuous semigroup generated by infinitesimal  $\mathcal{R}$  in Banach space  $X$  with norm  $\|\cdot\|$ , and  $D^\alpha$  is Riemann – Liouville fractional derivative,  $K: [0, b] \times X \rightarrow X$  is given function.  $X$  is a Banach space contains  $z(\cdot)$  Values, also  $L^2([0, b], \mathbb{U})$  is a Banach space,  $B$  is a bounded linear operator from  $\mathbb{U}$  into  $X$  and the control functions  $u(\cdot)$  in  $L^2([0, b], \mathbb{U})$ .

#### Definition (3.1):

The system (3.1) has a solution  $z: [0, b] \rightarrow X$  if satisfied:

$$z(t) = \mathcal{R}(t)z_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)Bu(s)ds.$$

#### Definition (3.2):

If there exists  $u \in L^2([0, b], \mathbb{U})$  is control as,  $\forall z_0, z_1 \in X$ , the solution of (3.1)  $z(t)$  satisfies  $z(b) = z_1$ , then (3.1) is controllable on  $[0, b]$ .

In order to demonstrate the main theorem of this section, we give the following hypotheses.

(1)  $\mathcal{R}(t)$  is a compact semigroup of bounded linear operator generated by the infinitesimal operator  $(\mathcal{R})$ , and  $\max_{0 \leq t \leq b} \|\mathcal{R}(t)\| = M$

(2)  $B: L^2(0, b; \mathbb{U}) \rightarrow L^1(0, b; X)$  is bounded linear operator from a Banach space  $\mathbb{U}$  into  $X$ .

(3)  $W: L^2(0, b; \mathbb{U}) \rightarrow X$  is a linear operator defined by.  $Wu = \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)(Bu)(s)ds$ , and the operator  $W^{-1}$  is inverse of  $W$ .

(4) .

(i)  $f: [0, b] \rightarrow X$  is strongly measurable.

(ii)  $\sup_{0 \leq t \leq a} \|f(t, x)\| \leq ga \in L^1(0, b)$ , For each positive  $(a)$  and.

$$\lim_{a \rightarrow \infty} \frac{1}{a} \int_0^b ga(s)ds = \alpha < \infty, \alpha \text{ is real number.}$$

$$(5) \quad \alpha M \tilde{M} [1 + M \tilde{M} \|B\| \|W^{-1}\|] < 1$$

#### Theorem (3.3):

The system (3.1) is controllable on  $[0, b]$ , if the conditions (1-5) are holds.

#### Proof:

Define

The operator  $P: C(0, b; X) \rightarrow C(0, b; X)$  as,

$$(Pz)(t) = \mathcal{R}(t)z_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)Bu(s)ds, \quad \text{for each } z_0 \in (0, b, X) \quad (3.2).$$

Then by using condition (3) for any  $z(\cdot)$ , define the control

$$u(t) = W^{-1} \left[ z_1 - \mathcal{R}(t)z_0 - \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(b-s)H(s, z(s))ds \right] (t)$$

Next, by using  $u(t)$  to show  $z$  as a fixed point of  $(P)$ , so that this point is a solution of system (3.1). Therefore clear that

$$\begin{aligned} (Pz)(b) &= \mathcal{R}(t)z_0 + \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)H(s, z(s))ds \\ &\quad + \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)BW^{-1}[z_1 - \mathcal{R}(b)z_0 \\ &\quad - \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)H(\tau, z_\tau)d\tau] ds = z_1 \end{aligned}$$

$$\mathcal{R}(b-s)H(\tau, z_\tau)d\tau] ds = z_1.$$

Now, we prove the operator  $(P)$  is continuous and convex, firstly continuously of  $P$ , so, let  $z_n \rightarrow z$  in  $C(0, b; X)$ ,

$$\text{Since } \|H(s, z_n(s)) - H(s, z(s))\| \leq ga(s) \text{ and}$$

$$\|H(\tau, z_{\tau_n}) - H(\tau, z_\tau)\| \leq ga(t), \text{ then}$$

$$H(s, z_n(s)) \rightarrow H(s, z(s)), \text{ as } n \rightarrow \infty$$

$$H(\tau, z_{\tau_n}) \rightarrow H(\tau, z_\tau), \text{ as } n \rightarrow \infty$$

Hence,

$$\|Pz_n - Pz\| = \sup_{0 \leq t \leq b} \left\| \mathcal{R}(t)z_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s) \{H(s, z_n(s)) - H(s, z(s))\} ds \right\|$$

$$\begin{aligned} &\sup_{0 \leq t \leq b} \left\| \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)BW^{-1} \left[ z_1 - \mathcal{R}(t)z_0 \right. \right. \\ &\quad \left. \left. - \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)H(\tau, z_{\tau_n}) - H(\tau, z_\tau) d\tau \right] ds \right\| \rightarrow 0 \end{aligned}$$

as  $n \rightarrow \infty$ , thus  $P$  is continuous.

Now, let  $B_a = \{z \in C(0, b; X) : \|z(t)\| \leq a, a \in [0, b]\}$  for each  $(a)$  positive number, so we note that the set  $B_a$  is bounded, closed and convex in  $C(0, b; X)$  and the operator  $(p)$  is well defined on  $B_a$ , therefore suppose there exist appositve number  $(A)$  and we claim that  $P(B_a) \subseteq B_a$ , thus in order to prove this statement, we let for each natural number  $(a)$  there is  $(Z_a)$  a function such that,  $Pz_a \notin B_a$  and  $a \leq \|Pz_a\|$ , So,  $a < \|Pz_a(t)\|$ , for some  $t(a) \in [0, b]$  therefore,  $1 < \frac{1}{a} \|Pz_a\|$ ,

$$\text{then } 1 \leq \lim_{a \rightarrow \infty} \frac{1}{a} \|Pz_a\|,$$

and we get

$$\begin{aligned} \lim_{a \rightarrow \infty} \frac{1}{a} \|Pz_a\| &\leq \lim_{a \rightarrow \infty} a^{-1} \left\{ \|\mathcal{R}(t)z_0\| + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \|\mathcal{R}(t-s)\| \|H(s, z(s))\| ds + \right. \\ &\quad \left. \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \|\mathcal{R}(t-s)\| \|B\| \|W^{-1}\| \left[ \|z_1\| + \|\mathcal{R}(t)z_0\| + \right. \right. \\ &\quad \left. \left. \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \|\mathcal{R}(b-s)\| \|H(\tau, z_\tau)\| d\tau \right] ds \right\} \end{aligned}$$

$$\begin{aligned}
& \lim_{a \rightarrow \infty} a^{-1} \|Pz_a\| \\
& \leq \lim_{a \rightarrow \infty} \left\{ M \|z_0\| \frac{Mb^\alpha}{\Gamma(\alpha+1)} \int_0^b g_a(s) ds \right. \\
& \quad + \frac{Mb^\alpha}{\Gamma(\alpha+1)} \|B\| \|W^{-1}\| \left[ \|z_1\| + \|\mathcal{R}(t)z_0\| \right. \\
& \quad \left. \left. + \frac{Mb^\alpha}{\Gamma(\alpha+1)} g_a(\tau) d\tau \right] ds \right\} \\
& = \alpha M \tilde{M} + M \tilde{M} \|B\| \|W^{-1}\| M \tilde{M} \\
& \propto M \tilde{M} [1 + M \tilde{M} \|B\| \|W^{-1}\|] < 1 \text{ (by condition 5)}
\end{aligned}$$

But by condition (4) (ii), we have  $\lim_{a \rightarrow \infty} a^{-1} \|Pz_a\| \geq 1$ , so we obtain  $Pz_a \notin B_a$  is not true, there  $PB_A \subseteq B_A$  for some positive A.

Now we define and prove the set  $V(t) = \{Pz(t) : z \in B_A\}$  is precompact in X. Firstly  $V(o)$  is precompact in X. So, let  $0 \leq t \leq b$  and  $0 \leq t \leq b, \epsilon$  is a given real number.

#### Define

$$\begin{aligned}
(P_{\epsilon}z)(t) &= \mathcal{R}(t)z_0 + \frac{1}{\Gamma(\alpha)} \int_0^{t-\epsilon} (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds \\
& \quad + \frac{1}{\Gamma(\alpha)} \int_0^{t-\epsilon} (t-s)^{\alpha-1} \mathcal{R}(t-s)Bw^{-1}[z_1 \\
& \quad - \mathcal{R}(t)z_0 - \frac{1}{\Gamma(\alpha)} \int_0^b (b-s)^{\alpha-1} \mathcal{R}(b-s)H(\tau, z_\tau) d\tau] ds
\end{aligned}$$

For the compactness of  $\mathcal{R}(t)$  and  $u(s)$  is bounded, we obtain that  $V_{\epsilon}(t) = \{(P_{\epsilon}z)(t) : z \in B_A\}$  is precompact set in X.

Moreover, for  $z \in B_A$  and by defined control  $u(t)$ , we have

$$\begin{aligned}
\|(Pz)(t) - (p_z)_{\epsilon}(t)\| &= \|\mathcal{R}(t)z_0 \\
& + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds \\
& + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s)Bu(s)ds \\
& - \mathcal{R}(t)z_0 \\
& - \frac{1}{\Gamma(\alpha)} \int_0^{t-\epsilon} (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds \\
& - \frac{1}{\Gamma(\alpha)} \int_0^{t-\epsilon} (t-s)^{\alpha-1} \mathcal{R}(t-s)Bu(s)ds\|
\end{aligned}$$

$$\begin{aligned}
& \leq \left\| \frac{1}{\Gamma(\alpha)} \int_{t-\epsilon}^t (t-s)^{\alpha-1} \mathcal{R}(t-s)H(s, z(s))ds \right\| \\
& \quad + \left\| \frac{1}{\Gamma(\alpha)} \int_{t-\epsilon}^t (t-s)^{\alpha-1} \mathcal{R}(t-s)Bu(s)ds \right\| \\
& \leq \frac{M \epsilon^\alpha}{\Gamma(\alpha+1)} \int_{t-\epsilon}^t \|H(s, t(s))\| ds \\
& \quad + \frac{M \epsilon^\alpha}{\Gamma(\alpha+1)} \|B\| \|w^{-1}\| \int_{t-\epsilon}^t [\|z_1\| \\
& \quad + M \|z_0\| + \frac{Mb^\alpha}{\Gamma(\alpha+1)} \int_0^b \|H(t, z)\| d_s \|H(\tau, z_\tau)\| \tau d\tau] ds \\
& \leq \frac{M \epsilon^\alpha}{\Gamma(\alpha+1)} \int_{t-\epsilon}^t g_A(s) ds \\
& \quad + \frac{M \epsilon^\alpha}{\Gamma(\alpha+1)} \|B\| \|w^{-1}\| \int_{t-\epsilon}^t [\|z_1\| + M \|z_0\| + \frac{Mb^\alpha}{\Gamma(\alpha+1)} \int_0^b g_A(\tau) d\tau] ds
\end{aligned}$$

Thus, there are precompact sets close to the set  $V(t)$ .

Hence, we obtain that  $V(t)$  is also precompact in X.

Now to prove  $PB_A = \{Pz \in B_A\}$  is an equicontinuous family functions. To do this, let  $t \in B_A$  and  $t, t^* \in [o, b]$  such that  $o < t < t^*$ , so we have,

$$\begin{aligned}
\|(Pz)(t) - (Pz)(t^*)\| &\leq \|\mathcal{R}(t) - \mathcal{R}(t^*)\| \|z_0\| + \\
& \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s) - (t^*-s)^{\alpha-1} \mathcal{R}(t^*-s) \\
& \|H(s, z(s))\| ds + \frac{1}{\Gamma(\alpha)} \int_0^{t^*} (t^*-s)^{\alpha-1} \mathcal{R}(t^*-s) \\
& \|H(s, z(s))\| ds + \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} \mathcal{R}(t-s) - \\
& (t^*-s)^{\alpha-1} \mathcal{R}(t^*-s) \|B\| \|w^{-1}\| [\|z_1\| + \|\mathcal{R}(b)\| \|z_0\| + \\
& \frac{1}{\Gamma(\alpha)} \int_0^b \|(b-s)^{\alpha-1} \mathcal{R}(b-s)\| \|H(\tau, z_\tau)\| d\tau] ds + \\
& \frac{1}{\Gamma(\alpha)} \int_t^{t^*} \|(t^*-s)^{\alpha-1} \mathcal{R}(t^*-s) - \\
& (t-s)^{\alpha-1} \mathcal{R}(t-s)\| [\|z_1\| + \|\mathcal{R}(b)\| \|z_0\| + \\
& \frac{1}{\Gamma(\alpha)} \int_0^b \|(b-s)^{\alpha-1} \mathcal{R}(b-s)\| \|gk(\tau)\| d\tau] ds + \\
& \frac{1}{\Gamma(\alpha)} \int_t^{t^*} \|(t^*-s)^{\alpha-1} \mathcal{R}(t^*-s)\| \|gk(s)\| ds \frac{1}{\Gamma(\alpha)} \int_0^t \|(t-s)^{\alpha-1} \mathcal{R}(t-s) - \\
& (t^*-s)^{\alpha-1} \mathcal{R}(t^*-s)\| \|B\| \|w^{-1}\| [\|z_1\| + \|\mathcal{R}(b)\| \|z_0\| + \\
& \frac{1}{\Gamma(\alpha)} \int_0^b \|(b-s)^{\alpha-1} \mathcal{R}(b-s)\| \|gk(\tau)\| d\tau] ds + \\
& \frac{1}{\Gamma(\alpha)} \int_t^{t^*} \|(t^*-s)^{\alpha-1} \mathcal{R}(t^*-s)\| \|B\| \|w^{-1}\| [\|z_1\| + \\
& \|\mathcal{R}(b)\| \|z_0\| + \frac{1}{\Gamma(\alpha)} \int_0^b \|(b-s)^{\alpha-1} \mathcal{R}(b-s)\| \|gk(\tau)\| d\tau] ds
\end{aligned}$$

Therefore, we note that  $g_A(t) \in L^1[0, b]$  and we see, as  $t \rightarrow t^*$ , holds

$\|(Pz)(t) - (Pz)(t^*)\|$  tends to zero and that means the right hand also goes to zero, and  $\mathcal{R}(t)$  is continuous because it is compact.

Hence  $(PB_A)$  maps  $B_A$  in to a family of equicontinuous functions and also bounded.

Then the operator  $(PB_A)$  is precompact in  $C(0, b; X)$  by Arzela – Ascoli theorem. So "P" is completely continuous in  $C(0, b; X)$  by Arzela – Ascoli theorem.

Therefore by fixed point theorem of Schauder, "P" has a fixed point in  $(B_A)$ , and this point is a solution of

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(3.1) that is satisfying  $(P_z)(t) = z(t) \in X$ .  
Hence, the system (3.1) is controllable on  $[0, b]$ .

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## قابلية السيطرة لحل المعادلات التفاضلية التكاملية الكسرية الشبه خطية في فضاء بناخ

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### المخلص

الهدف من الدراسة هو برهان قابلية السيطرة للحل المعطول للمعادلات التفاضلية التكاملية الكسرية مع شروط غير محلية في فضاء بناخ. التفاضل الكسري، شبه المجموعات المتراسة ونظرية النقطة الصامدة مفاهيم تم الاستعانة بها للحصول على نتائج هذا العمل.