TJPS

TIKRIT JOURNAL OF PURE SCIENCE

Journal Homepage: http://main.tu-jo.com/ojs/index.php/TJPS/index

Some Properties of the Members of Petersen Family

Israa M. Tawfik

Mathematics Department, College of Education for Women, Tikrit University, Iraq.

ARTICLE INFO.

Article history:
-Received: 16 / 10 / 2017
-Accepted: 21 / 12 / 2017
-Available online: / / 2018

Keywords: Petersen graph, complete graph, K_6 graph, cut vertex.

Corresponding Author:
Name: Israa M. Tawfik
E-mail: Imt011@yahoo.com
Tel:

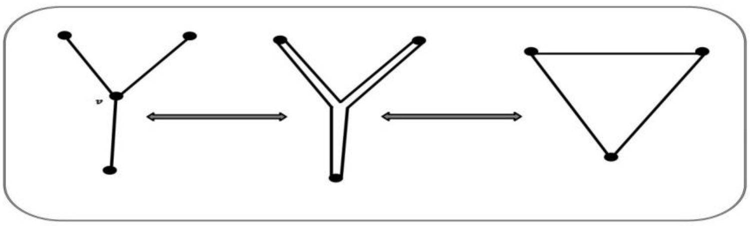
Abstract

This article has restudied the Petersen family in Graph Theory. It discussed the process of establishing this family. This discussion leads to discovering some new properties of the Petersen family's members.

1. Introduction

We derive some fundamental properties of members of an exciting set of seven graphs called Petersen family. The idea of this research came from our attractive to use this family in topological surfaces; therefore, we decided to restudy this family. This study leads to new some significant and simple results.

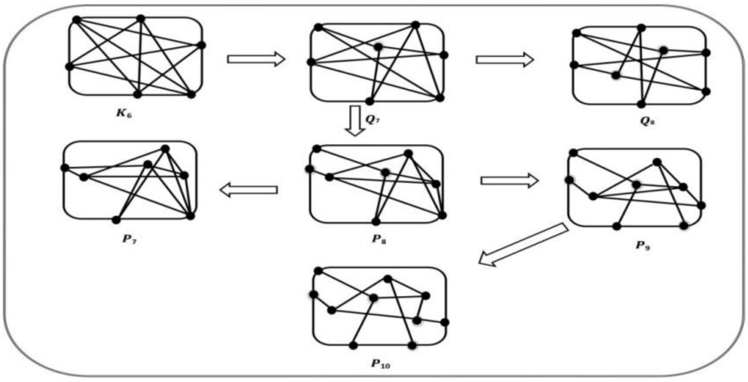
Petersen family constructs from K_6 by $\nabla \leftrightarrow Y$ exchanges. If $G = (V, E)$ is a connected graph having V a set of vertices and E a set of edges in G (each edge connects two vertices, these vertices called end-vertices). Then $\nabla \to Y$ exchange is an operation obtained from G by deleting the three edges of the triangle and add a new vertex v to be the adjacent vertex of each vertex of the triangle, v having degree 3 (it is defined later), v called the center of the Y. Now, we have a new graph $H = (W, F)$ as in Figure 1. The converse of this deal is called $Y \to \nabla$ exchange [1]. K_6 is a complete graph (it is defined later), that means each three vertices in it are consistence a triangle. The changing of any triangle in K_6 gives a graph Q_7, (We use the usual name for the Petersen family's members). Any triangle in K_6 has a disjoint triangle. When we choose it in Q_7 and change it to Y, we get a graph Q_8. Q_8 does not have any triangle. To get P_8, we can choose any other triangle in Q_7. This triangle shares the exchanging triangle in Q_7 a vertex or an edge. This operation makes an original vertex (appears in K_6) now in P_8 of degree 3. This vertex adjacent to the new vertices appears in P_8. That means P_8 has another Y when this Y is changed to ∇; a P_7 graph is constructed. P_8 has two disjoint triangles, then choosing one of them to change it to Y gives a P_9 member in the Petersen family. A final graph in this family is P_{10} obtained from the graph P_9. P_{10} is called the Petersen graph [2].



Figure(1): $\nabla \leftrightarrow Y$ exchange.

There are many previous studies on this subject, where [1] used the embedding of Petersen family. [2] worked with the Petersen graph, they talked about the history of this graph and used it on different sides in graph theory. [3] Found some outstanding results used ∇↔Y exchanges for different types of graphs when they constructed graphs and binary matroids of graphs. [5] got more e forbidden minors for wye-delta-wye reducibility. [5] linked between Graph Theory and Knot Theory by using this family.

[6] Show that a graph is obtained from the complete graph on seven vertices by a finite sequence of ∇↔Y exchanges was a minor-minimal intrinsically knotted or completely 3-linked graph. [7] and [8] constructed subfamilies of Petersen using the embedding of K_6 in the torus and Klein bottle sequentially.



Figure(2): The scheme of Petersen Family.

We have discussed the construction of the Petersen family in the first section of this work. The second section contains of some definitions, that we need them in this research and new theorems for some meaningful results. The third section is the main part of this work. This section consists of results are sometimes properties for all members of this family or some members.

2. Primary Results

In this section, we have some fundamental definitions and new achievements to use them in the central section of this article.

First, we have to mention the construction of the Petersen family that has been discussed in the introduction.

To define a connected graph we have to refer to a walk which is a sequence of edges and vertices, where a path is a walk without repeat the vertex [9].

Definition 1 [10]: A graph G is called a connected graph if it has a path between each pair of vertices in G. Otherwise is referred to as a disconnected graph.

Definition 2 [11]: A graph G is called a complete graph if it is a connected graph and has one edge between each pair of vertices in G. It is known K_n, where n is the number of vertices.

From the qualifier of $\nabla \leftrightarrow Y$ exchanges and the above definitions, we get the following theorem.

Theorem 1: The connectedness is keeping under $\nabla \leftrightarrow Y$.

Proof

Since Y has one path between each pair of it is vertices, and any triangle is K_3. That means the operation of $\nabla \leftrightarrow Y$ exchanges keeps the connectedness. □

Any edge of a graph is called a loop if it has the same end-vertices. A pair of edges is called parallel edges if these two edges have the same end-vertices [9].

Definition 3 [12]: A graph G is called a simple graph if it has no loops or parallel edges.

Definition 4 [9]: If F is a set of edges in G, and the deletion of this set makes a new graph $G - F$ a disconnecting graph, then F is called a disconnecting set in G. If F contains one edge then this edge is called a bridge. If F consists of at least k edges, then G is called a k- disconnected graph.

Definition 4 leads to the following obvious theorem.

Theorem 2: Any complete graph K_n is an $(n-1)$-disconnected graph.

Proof

Suppose $G = K_n$ that means each vertex in G joins with $n-1$ vertices by edges. The deletion of these edges makes G a disconnecting graph. Hence, the result holds. □

K_6 is a 5-disconnected graph.

Definition 5 [13]: A degree of a vertex $v \in G$ is the number of edges pass from v. It is denoted by $d(v)$. The size of G is the number of E's elements in G.

Theorem 3: Any graph contains Y is a 3-disconnected graph.

Proof

Suppose G is a graph contains Y, and then G has a vertex v of degree 3. The deletion of all edges pass from v makes G a 3-disconnected graph. □

Definition 6 [12]: A graph G is called a k-regular graph if the degree of each vertex in G is k.

Definition 6 leads to the following obvious theorem.

Theorem 4: Any complete graph K_n is an $(n-1)$-regular graph.

K_6 is a 5-regular graph.

Definition 7 [9]: A vertex v in a graph G is called a cut vertex if the deletion of v makes a new graph $G - v$ disconnecting graph.

Theorem 5: Any complete graph K_n has not a cut vertex.

ISSN: 1813 – 1662 (Print)
E-ISSN: 2415 – 1726 (On Line)

Proof

Suppose $G = K_n$, and G has a cut vertex that means there are at least two vertices which do not join by an edge. This case is a contradiction to the definition 2.

Definition 8 [13]: A vertex v in a graph G is called an isolated vertex if $d(v) = 0$. A graph G is called a trivial graph if $d(v) = 0$ for each v in G.

Definition 9 [9]: Graphs $G = (V, E)$ and $H = (W, F)$ are called isomorphism if there is a one-one function from V to W. Such that if there is an edge between v_1 and v_2 in G that means there is an edge between $f(v_1)$ and $f(v_2)$ in H.

3.Some Properties of Petersen Family's Members:

Theorem 6: Each graph in the Petersen family is:

1- a connected graph.

2- is a simple graph.

Proof

1- Since K_6 is a complete graph, then it is a connected graph by definition 2. Then by Theorem 1 each other member in the Petersen family is a connected graph.

2- By definition 2 and 3, K_6 is a simple graph. $\nabla \leftrightarrow Y$ exchanges do not add a loop or parallel edges to any graph. Hence, the theorem holds. □

Now, we discuss some properties of some members separately.

Theorem 7: Q_7, Q_8, P_8, P_9 and P_{10} are 3-disconnected graphs.

Proof

Every graph has a Y. Then by Theorem 3, this theorem is satisfied. □

Theorem 8: P_7 is a 4-disconnected graph.

Proof

P_7 is a graph constructed from P_8 by $Y \to \nabla$ exchange. Suppose v is the centre of Y (a vertex of degree 3) in P_8, where $v \in K_6$. Also, $d(x_1) = 3$ and $d(x_2) = 3$, x_1 and x_2 are the two end-vertices of the edges pass from v, where $x_1 \in Q_7$and $x_2 \in P_8$. The changing this Y to ∇ increases one the degree of x_1 and x_2. The theorem holds. □

Theorem 9: The Petersen family does not contain a member has:

1- A bridge.

2- A cut vertex.

Proof

1- Suppose G is a one member in Petersen family has a bridge. What that means G is a 1-disconnected graph, but this is a contradiction to Theorems 4, 7, and 8.

2- If G is a member of the Petersen family and G has a cut vertex. $G \neq K_6$ From Theorem 5. If $G = Q_7$, that means this cut vertex is a vertex in a Y has added to Q_7. Any vertex of the end-vertices of this Y is not this cut vertex by Definition 2. If the center of Y is this cut vertex, that leads to there are two vertices in G (maybe one of them is the end-vertex of Y), these two vertices do not connect by an edge, but this is a contradiction to Theorem 5. Using the same discussion of this proof, we can prove no any member in the Petersen family has a cut vertex to completes this proof. □

Corollary: The Petersen family has no member as a trivial graph, and no member has an isolated vertex.

Theorem 10: There are no two members are isomorphic in the Petersen family.

Proof

This theorem is satisfied from the construction of this family. □

4- Conclusions and further work:

This article discussed the constructing of the Petersen family. Also found properties of some kinds of graphs relevant to the members of the Petersen family. Then proved particular theorems of the members of the Petersen family.

This work can be developed by trying to find other properties of these members. This operation complete by studying this family from other aspects such as the Eulerian Graph or the Hamiltonian Graph. The studying of these two types means research on the subject of circuits and cycles of graphs. The above properties can be explored by converting these graphs into matrices.

It should be noted that the first author of this article has tried using advanced programs to convert these elements to polynomials using Tutte Polynomial. Meanwhile, the first author has started to study the embedding of this family in different topological surfaces since 2012.

References

[1] Robertson, N. (1995). Petersen Family Minor, Journal of Combinatorial Theory, Series B64, P155-184.

[2] Holton, D., A. and Sheehn, J. (1993). The Petersen Graph, Cambridge University Press.

[3] Trumper, K. (1998). Matriod Decomposition, Leibniz Company.

[4] Yu, Y. (2006). More Forbidden Minors for Wye-Delta-Wye Reducibility, The Electronic Journal of Combinatorics, 13, P1-15.

[5] Flapan, E. and Naimi, R. (2008). The Y-Triangle Move Does Not Preserve Intrinsic Knottedness, Osaka J. Math 45, P107-111.

[6] Hanaki, R., Nikkuni, R., Taniyama, K. and Yamazaki, A. (2011). On Intrinsically Knotted or Completely 3-Linked Graphs, arxiv: 1006.0698 V4 [mathGT], P1-17.

[7] Tawfik, I. (2013). Links and Graphs, University of Plymouth.

[8] Tawfik, I., and Mahmoud, B.(2017). Constructing Some Embedded of Petersen Subfamilies in The Klein Bottle Using Link Diagram, Global Proceeding Repository, American Research Foundation, P528-538.

[9] Balarishnan, V.K. (1997). Schaum's Outline of Graph Theory, McGraw-Hill.

[10] Bollobas', B. (2002). Modern Graph Theory, Springer-Verlag.

[11] Diestel, R. (2010). Graph Theory, Springer.

ISSN: 1813 – 1662 (Print)
E-ISSN: 2415 – 1726 (On Line)

[12] Harary, F. (1971). Graph Theory, Addison Wesley.

[13] Ruohonen, K. (2013). Graph Theory, Tampere University of Technology Course.

بعض خصائص عناصر عائلة بترسون

اسراء منير توفيق

قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

الملخص

يقوم هذا البحث على اعادة دراسة عائلة بترسون في نظرية البيانات. كما ناقش البحث كيف يمكن انشاء هذه العائلة. هذه المناقشة قادتنا الى اكتشاف مواصفات جديدة لعناصر هذه العائلة. الكلمات المفتاحية:بيان بترسون، البيان الكامل، البيان K_6، الراس القاطع.