Tikrit Journal of Pure Science 23 (9) 2018

ISSN: 1813 – 1662 (Print)
E-ISSN: 2415 – 1726 (On Line)

TJPS

TIKRIT JOURNAL OF PURE SCIENCE

Journal Homepage: http://main.tu-jo.com/ojs/index.php/TJPS/index

THE FORGOTTEN INDEX OF CERTAIN EDGE-GLUING GRAPHS

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ARTICLE INFO.

Article history:

-Received: 3 / 4 / 2018

-Accepted: 14 / 6 / 2018

-Available online: / / 2018

Keywords: Forgotten Index, Cycle Graph, Wheel Graph, Complete Graph, Web Graph, Edge Gluing.

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Abstract

The forgotten index is defined as $F(G)=\sum_{u\in V(G)}(du)^3$. Where $d(u)$ is a degree of vertex u. In this paper, we study certain known graphs such as cycle, complete, wheel, and web graphs, then computed the forgotten index of their edge-gluing graphs.

1. Introduction

A topological index is a real number related to a graph, many topological indices find applications as mean to model chemical, pharmaceutical and other properties of molecules. Molecular descriptors have many applications in QSPR and QSAR studies [1, 2]. Many researchers studied and analysed topological indices for some graphs of chemical structures and nanostructures [3, 4, 5, 6]. One of the oldest and most popular index is the Zagreb index presented by Gutman and Trinajestic in 1972 [7]. Recently, Furtula and Gutman have introduced a new topological index called the forgotten index [8].

In 2014 an unexpected chemical application was discovered from the forgotten index, and it was proved that the forgotten topological index could greatly enhance the physical and chemical application of the first Zagreb index [9].Where it turned out there are strong internal relationships between bio-medical properties and pharmacology of drugs and their molecular structures. The forgotten topological index has been defined for use in the analysis of drug molecular structures, which is very useful for pharmaceutical and medical scientists to understand the biological and chemical properties of new drugs [10].

In 2013 Lin, considered that connected simple graphs are nontrivial. A graph is symbolized by $G=(V(G),E(G))$, where $V(G)$ is the vertex sets of G and $E(G)$ is the edge sets of G. If $v_iv_j \in E$, we denote to the graph acquired after removing the edge (v_i,v_j) from the G by $G-(v_i,v_j)$. If $(v_i,v_j)\notin E$, we denote to the graph acquired by adding the edge (v_i,v_j) to G by $G+(v_i,v_j)$ [11].

In 2015 Furtula and Gutman refer to the importance of forgotten index that it can be used to obtain a high accuracy of the prediction of logarithm of the octanol-water partition coefficient [8]. In 2016 Wei Gao, et. al. extend the forgotten topological index of several important chemical structures which have high frequency in drug structures [10]. Also in 2016 A. M. Khalaf, et. al. computed the atom bond connectivity index of some certain and vertex gluing graph [1]. In 2017 Elumalai. et al obtained, analyzed, and compared various lower bounds for the forgotten topological index involving the number of vertices, edges, maximum and minimum vertex degree [12]. Also in 2017, S. Ghobadi and M.Ghorbaninejad calculated the first Zagreb index, the forgetting index and the forgetting co-index of the line sub-division graphs [13]. Moreover In 2017 Hosam Abdo et. al. examined the trees extremal with respect to the forgotten index [14]. And In 2017 Shehnaz Akhter and Muhammad Imran studied the four operations of forgotten index on graphs and determine the closed formulas for the forgotten index of four operations on graphs. [2]. in in this work, we investigate the forgotten index of certain known graphs, moreover the edge-gluing of they obtained graphs by deriving general formulas.

ISSN: 1813 – 1662 (Print)
E-ISSN: 2415 – 1726 (On Line)

2. Basic definition

Definition 2.1. [15] The cycle graph is a graph obtained by merging the terminal vertices of the path graph. Thus, each vertex of a cycle graph is of degree two. A cycle graph with n vertices denoted by C_n, (Figure 1).

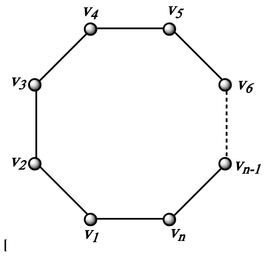

Fig.1. Cycle Graph C_n.

Definition 2.2. [16] The complete graph is a graph in which any two vertices are adjacent. The complete graph has $\frac{n(n-1)}{2}$ edges. We denote the complete graph on n vertices as K_n. (Figure 2).

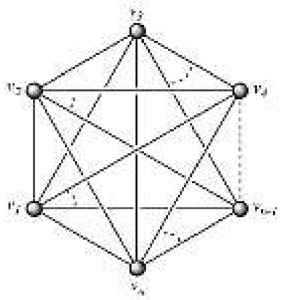

Fig.2. Complete Graph K_n

Definition 2.3. [17] The wheel graph denoted by W_n is a graph obtained from $C_{n-1} + K_1$. That is, the wheel graph is a cycle graph together with the complete graph K_1 at the center which is connected to all the vertices of C_n, (Figure 2).

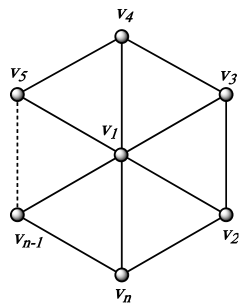

Fig.3.Wheel Graph W_n

Definition 2.4. [18] A Web graph $W_{n\times r}$ is a simple graph obtained from the Cartesian product graph $W_{n\times r} = C_n \times P_r$, where P_r is a path with r vertices (Figure 4).

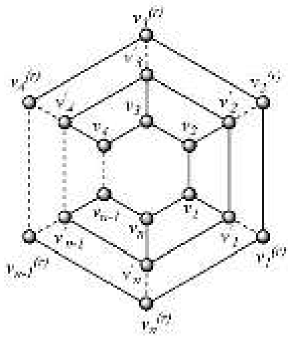

Fig.4. Web Graph $W_{n\times r}$

Definition 2.5. [5] Let $G_n$and G_m be graphs, each one of them containing a complete sub-graph K_P (p ≥ 1). Let G be a graph obtained from the union of G_n and G_m by merge the two sub-graphs K_P in any arbitrary way (Figure 5), then we call G a K_P-gluing of G_n and G_m and denoted by G_n^m.

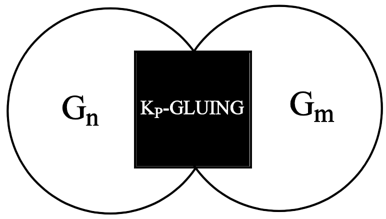

Fig.5.Gluing Graph G_n^m.

Definition 2.6. The e - gluing of graph G is a graph obtained from gluing two different graphs G_n and G_m by one edge.

Definition 2.7. [19] The e - gluing of cycle graph denoted by C_n^m is a graph obtained from gluing two different cycle graphs C_n and C_m by one edge $e_g = (g_1, g_2)$, where g_1 =(Fusion $v_1$with u_1) and g_2 = (Fusion v_2 with u_2), (Figure 8).

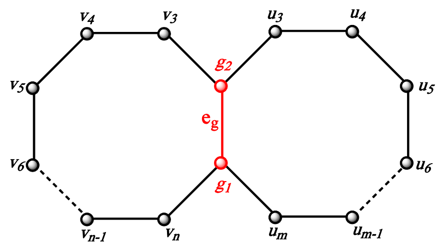

Fig.6. e-gluing Cycle Graph C_n^m.

Definition 2.8. The e - gluing of complete graph denoted by K_n^m is a graph obtained from gluing two different complete graphs K_n and K_m by one edge $e_g = (g_1, g_2)$, where g_1 =(Fusion $v_1$with u_1) and g_2 = (Fusion v_2 with u_2), (Figure 7).

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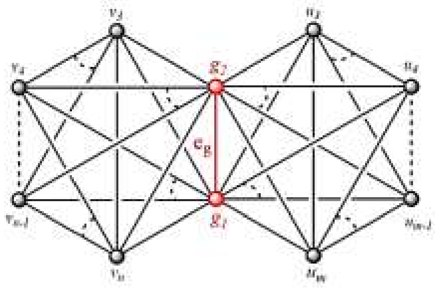

Fig. 7. e-gluing Complete Graph K_n^m .

Definition 2.9. The e – gluing of wheel graph denoted by W_n^m is a graph obtained from gluing two different wheel graphs W_n and W_m by one edge $e_g = (g_1, g_2)$, where g_1 =(Fusion $v_2$with u_2) and g_2 = (Fusion v_3 with u_3), (Figure 6).

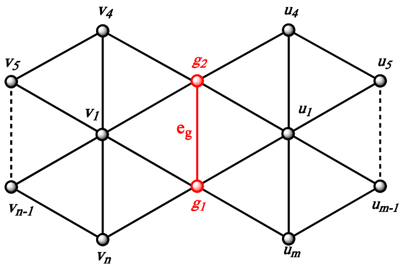

Fig.8. e-gluing Wheel Graph W_n^m .

Definition 2.10. The e - gluing of web graph denoted by $W_{n\times r}^{m\times s}$ is a graph obtained from gluing two different Web graphs $W_{n\times r}$ and $W_{m\times s}$ by one edge $e_g = (g_1, g_2)$, where g_1 = (Fusion v_1^r with u_1^s) and g_2 =(Fusion v_2^r with u_2^s), (Figure 9).

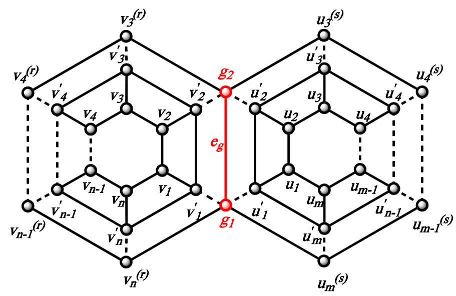

Fig.9. e-gluing Web Graph $W_{n\times r}^{m\times s}$.

3. The forgotten index of some special graphs

Lemma 3.1. [2] The forgotten index of a cycle graph C_n is

$F(C_n) = 8n$

Lemma 3.2. The forgotten index of a complete graph K_n is

$F(K_n) = n(n-1)^3$

Proof: The number of vertices in K_n are n all of degree $(n-1)$. Therefor by using definition of forgotten index we get

$F(K_n) = n(n-1)^3$

Lemma 3.3. The forgotten index of a wheel graph W_n is

$F(W_n) = n^3 - 3n^2 + 30n - 28$

Proof: The number of vertices in W_n are n. $(n-1)$ vertices each has degree 3 and one vertex has degree$(n-1)$. Therefor by using the definition of forgotten index we get

$F(W_n) = (n-1)(3)^3 + (1)(n-1)^3$

$= n^3 - 3n^2 + 30n - 28$

Lemma 3.4. Let n and r be positive integers. Then the forgotten index of a web graph $W_{n\times r}$ is given by:

$$F(W_{n\times r}) = \begin{cases} 64nr - 74n & if\ r > 2 \\ 54n & if\ r = 2 \end{cases}$$

Proof: If $r > 2$, the number of vertices in $W_{n\times r}$ are nr vertices. $2n$ vertices each has degree 3 and $n(r-2)$ vertices each has degree 4. Hence, by using the definition of forgotten index we get

$F(W_{n\times r}) = 2n(3)^3 + n(r-2)(4)^3$

$F(W_{n\times r}) = 64nr - 74n$

If $r = 2$, the number of vertices in $W_{n\times 2}$ are $2n$ vertices, each has degree 3.

by using the definition forgotten index we get

$F(W_{n\times 2}) = 2n(3)^3$

$= 54n$

4. The main ruselets

We proof by mathematical induction for more details and more clarity

Theorem4.1. Let n and m be positive integers numbers where $n, m \geq 3$, the forgotten index of e − gluing of cycle graph C_n^m is

$F(C_n^m) = 8n + 8m + 22$

Proof:

The number of vertices in C_n^m are $(n+m-2)$ vertices. $(n+m-4)$ vertices each has degree 2 and the remaining 2 vertices each has degree 3. Therefor by using the definition forgotten index, we get

$F(C_3^m) = (n+m-4)(2)^3 + 2(3)^3$

$= 8n + 8m - 32 + 54$

$= 8n + 8m + 22$

Theorem4.2. Let n and m be positive integers and $n, m \geq 3$, the forgotten index of e − gluing of complete graph K_n^m is

$F(K_n^m) = (n-2)(n-1)^3 + (m-2)(m-1)^3 + 2(n+m-3)^3$

Proof: The number of vertices in K_n^m are $(n+m-2)$ vertices. $(n-2)$ vertices each has degree $(n-1)$, $(m-2)$ vertices each has degree $(m-1)$ and 2 vertices each has degree $(n+m-3)$. Therefor by using the definition of forgotten index we get

$F(K_n^m) = (n-2)(n-1)^3 + (m-2)(m-1)^3 + 2(n+m-3)^3$

Theorem4.3. Let n and m be a positive integers and $n, m \geq 4$, the forgotten index of e − gluing of wheel graph W_n^m is

$F(W_n^m) = n^3 + m^3 - 3n^2 - 3m^2 + 30n + 30m + 86$

Proof: The number of vertices in W_n^m are $(n+m-2)$ vertices. $(n+m-6)$ vertices each has degree 3 , two vertices each has degree 5, one vertex has degree $(n-1)$ and one vertex has degree $(m-1)$.Therefor by using the definition of forgotten index we get

$F(W_n^m) = (n+m-6)(3)^3 + 2(5)^3 + (n-1)^3 + (m-1)^3$

$F(W_n^m) = 27n + 27m - 162 + 250 + n^3 - 3n^2 + 3n - 1 + m^3 - 3m^2 + 3m - 1$

$F(W_n^m) = 30n + 30m - 164 + 250 + n^3 - 3n^2 + m^3 - 3m^2$

$F\ (W_n^m) = n^3 + m^3 - 3n^2 - 3m^2 + 30n + 30m + 86$

Theorem4.4. Let n, m, r and s be positive integers and $n, m \geq 3$, the forgotten index of $e -$ gluing of web graph $W_{n\times r}^{m\times s}$ is

$$F(W_{n\times r}^{m\times s}) = \begin{cases} 54n + 54m + 142 & if\ r,s = 2 \\ 54n - 74m + 64ms + 142 & if\ r = 2, s > 2 \\ 64nr + 64ms - 74n - 74m + 142 & if\ r,s \neq 2 \end{cases}$$

Proof: If $r, s = 2$, there are $(2n + 2m - 2)$vertices in $W_{n\times 2}^{m\times 2}$ (Fig.15). $(2n + 2m - 4)$ vertices each has degree 3 and two vertices each has degree 5. Hence by using the definition of forgotten index, we get:

$F(W_{n\times 2}^{m\times 2}) = (2n + 2m - 4)(3)^3 + 2(5)^3$
$= 54n + 54m - 108 + 250$
$= 54n + 54m + 142$

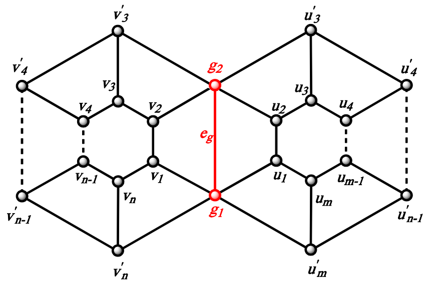

Fig.10.Graph $W_{n\times 2}^{m\times 2}$

If $r = 2$ and $s > 2$, there are $(2n + sm - 2)$ vertices in $W_{n\times 2}^{m\times s}$ (Fig.16).$(2n + 2m - 4)$ vertices each has degree3. Moreover $m(s - 2)$ vertices each has degree 4 and two vertices each has degree 5. Hence, we get:

$F(W_{n\times 2}^{m\times s}) = (2n + 2m - 4)(3)^3 + m(s - 2)\ (4)^3 + 2(5)^3$
$= 54n + 54m - 108 + 64ms - 128m + 250$
$= 54n - 74m + 64ms + 142$

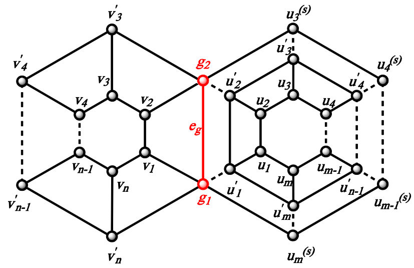

Fig.11.Graph $W_{n\times 2}^{m\times s}$

If $r, s \neq 2$, there are $(rn + sm - 2)$ vertices in $W_{n\times 2}^{m\times s}$(Fig.1-10). $(2n + 2m - 4)$ vertices each has degree 3. Moreover $n(r - 2)$ vertices each has degree 4, $m\ (s - 2)$ vertices each has degree 4 and the remaining two vertices each has degree 5. Hence, we get

$F(W_{n\times r}^{m\times s}) = (2n + 2m - 4)(3)^3 + n(r - 2)\ (4)^3 + m(s - 2)\ (4)^3 + 2(5)^3$
$= 54n + 54m - 108 + 64nr - 128n + 64ms - 128m + 250$
$= 64nr + 64ms - 74n - 74m + 142$

5. Conclusion

In this paper, we presented the general formulas of forgotten index for known graphs, wheel, complete, cycle and web graphs then give formulas in general of these graphs when we glue the two same type of graphs by one edge.

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مؤشر الفوركوتن لرسوم بيانية معينة ملتصقة بحافة

احمد ماهر صالح ، نبيل عزالدين عارف

قسم الرياضيات ، كلية علوم الحاسوب والرياضيات ، جامعة تكريت ، تكريت ، العراق

الملخص

مؤشر الفوركوتن معرف على انه مجموع لتكعيب جميع درجات الرؤوس لرسم بياني بمعنى $F(G)=\sum_{u\in V(G)}(du)^3$ حيث du تمثل الـ درجة الرأس u ،في بحثنا هذا قمنا بدراسة بعض الرسوم البيانية الخاصة مثل بيان الدارة ، البيان الكامل ،بيان العجلة و بيان الشبكة . ومن ثم استخرجنا صيغة عامة لمؤشر الفوركوتن لهذه الرسوم البيانية عند لصقها بحافة واحدة .